Anomalous transmission in a hierarchical lattice

Anirban Chakraborti*
Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Calcutta 700 064, India

Bibhas Bhattacharyya†
Department of Physics, Scottish Church College, 1 & 3 Urquhart Square, Calcutta 700 006, India

Arunava Chakrabarti‡
Department of Physics, University of Kalyani, Kalyani, West Bengal 741 235, India
(Received 14 July 1999)

We present an analytical method of studying ''extended'' electronic eigenstates of a diamond hierarchical lattice, which may be taken as the simplest of the hierarchical models recently proposed for stretched polymers. We use intuitive arguments and a renormalization-group method to determine the distribution of amplitudes of the wave functions corresponding to some of these ''extended'' eigenstates. An exact analysis of the end-to-end transmission property of arbitrarily large finite lattices reveals an anomalous behavior. It is seen that while for a special value of the energy the lattice, however large, becomes completely transparent to an incoming electron, for the other energy eigenvalues the transmission decreases with system size. For one such energy eigenvalue we analytically obtain the precise scaling form of the transmission coefficient. The same method can easily be adopted for other energies.

I. INTRODUCTION

Hierarchical lattice models have played an important part in understanding the statistical mechanics of phase transitions. Such lattices differ, both in topology and in geometry, from the Bravais lattices.[1] One major feature of such structures is their scale invariance, which enables an exact implementation of real-space renormalization-group (RSRG) techniques.[1–4] Apart from statistical mechanics, studies of the electronic spectrum of such lattices have also revealed striking properties, not shared by common crystalline structures.[5–7] For example, Domany *et al.*[5] solved the Schrödinger equation on a variety of hierarchical lattices using an exact recursive scheme. In the thermodynamic limit, the energy levels were found to be discrete, very closely spaced and highly degenerate.[5] The electron localization problem however, is not easy to understand on hierarchical lattices. The fluctuating local environment around each lattice point is likely to localize the electronic wave functions in most situations, though for some hierarchical or fractal structures like the Seirpinski gasket and the Vicsek fractal, ''extended'' electronic states have been reported.[8,9] This aspect makes the study of electronic properties of hierarchical lattices interesting. Normally, the ''extended'' character of electronic wavefunction is associated with translational invariance of the underlying lattice and, a hierarchical lattice (like the present one) does not have any translational periodicity by virtue of its construction (see Fig. 1). However, we are familiar with several one dimensional examples, viz. the one-dimensional random dimer[10] or quasiperiodic lattice models[11] where one comes across situations in which local positional correlation between constituent ''atoms'' gives rise to a finite[10] or infinite[11] number of ''extended'' electronic states even though these lattices are not periodic. In hierarchical lattices, such local positional correlation is not always obvious. On the other hand, the topology of the lattice in certain earlier cases has been shown to play an important role in sustaining ''extended'' electronic wave functions.[8,9] Such states sometimes exhibit anomalous transport properties, in the sense that though the amplitude of the wave function remains nonzero even at distant parts of arbitrarily large lattices, the end-to-end transmission of any incident wave packet sometimes displays a power-law decay.[9]

Above exact results are available for hierarchical models in which there is a finite variety of the nearest-neighbor environment around a lattice site (strictly speaking, in any hierarchical lattice all sites are inequivalent if one looks beyond the nearest neighbors). The situation may thus become quite challenging if one tries to explore a case where the range of the coordination number of the lattice points increases with the generation of bigger and bigger lattices. One such example is the well-known diamond hierarchical lattice[1] in which the coordination number of the vertices range from $z=2$ to $z_{max}=2^{N-1}$ for an Nth generation lattice. The thermodynamics of spin models on a diamond lattice has been studied exactly by RSRG methods.[1] But its electronic properties have not been really studied in detail, a part of which we intend to address in the present communication. Our interest in the diamond lattice is twofold. First, we wish to investigate analytically, if there exists any ''extended''

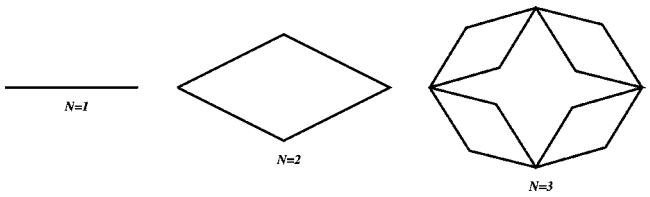

FIG. 1. First three stages (i.e., $N=1,2,3$) of the construction of a diamond hierarchical lattice.

electronic eigenstate though there is no translational periodicity in this lattice. Moreover, if such states do exist then, to our mind, it would be interesting to study their amplitude profiles and also to study the transmission properties of arbitrarily large finite lattices at those special values of the electron-energy for which the distribution of the amplitudes is non-trivial. Second, we note that similar hierarchical structures have recently been proposed by Samukhin *et al.*[12] as a possible basic structure of stretched polymers. In stretched polyacetylene the polymer network is constructed by coupled polymer chains oriented along some direction.[12] The (m,n) hierarchical pseudolattice that serves as the model for such polymers may be composed, according to Ref. 12, by taking n bonds forming a chain of $(n+1)$ atoms and then joining m such chains in parallel. It is then quite obvious that our diamond hierarchical structure is a (2,2) model lattice in this group (see Fig. 1). So, an analytical approach to study the transport properties of a diamond structure seems to be an interesting step towards the understanding of the properties of the general (m,n) structure.

Recently, Zhu *et al.*[13] numerically calculated the transmission coefficient of the (m,n) polyacetylene model as a function of electron energy for different (m,n) values and at different stage numbers. Interestingly, they found, among very fragmented patterns, many energy values for which the transmission coefficient turns out to be unity (or very nearly so) for up to fifth-generation structures. Also, they have reported (numerically obtained) scaling behavior of conductance for the (m,n) structures, though its precise form is not known. In view of this, we consider the simplest (2,2) version, i.e., the diamond lattice and make an analytical attempt to see if such a structure ever becomes completely or partially transparent to an incoming electron. Most interestingly, we find that it is possible to get a special energy eigenvalue for which arbitrarily large-sized diamond hierarchical lattices will have transmission coefficient equal to *unity*. We work out a scheme based on an intuitive approach to determine the distribution of the amplitudes of the wave function (ψ_i) for this special energy on a small sized lattice and then use an RSRG approach to work out the distribution in lattices for higher generations. The central idea is not too difficult to extend to the general (m,n) case. Using the same RSRG formalism we evaluate a whole hierarchy of energy eigenvalues for which we again have nontrivial distribution of the amplitudes ψ_i over the entire lattice. The end-to-end transmission across lattices of gradually increasing size is now found to decay for this second group of eigenvalues. We explicitly calculate the scaling behavior for one such case and obtain a precise form of the power law followed by the transmission coefficient. Scaling forms for other energies can be obtained following the scheme prescribed by us. In Sec. II, we describe our model and the method, in Sec. III the transmission coefficient is discussed, and we draw conclusions in Sec. IV.

II. THE MODEL AND THE METHOD

In Fig. 1, we show the first three stages of construction of the diamond lattice following Berker and Ostlund.[2] A detailed discussion on the construction and topology of lattices belonging to this class is given in Ref. 1. The atomic sites sit at every vertex of the lattice. To study the electronic properties of the diamond lattice, we work with the tight-binding Hamiltonian with nearest-neighbor approximation:

$$H=\sum_i \epsilon_i|i\rangle\langle i|+\sum_{\langle ij\rangle} t_{ij}|i\rangle\langle j|, \tag{1}$$

where ϵ_i is the on-site potential and t_{ij} is the nearest-neighbor (*NN*) hopping integral.

As we are primarily interested in the topological aspect of the system we set identical values to all the site energies, i.e., $\epsilon_i=\epsilon$ and all the hopping integrals are taken to be the same, i.e., $t_{ij}=t$ (chosen to be the scale of energy). However, to facilitate the renormalization group calculations we shall designate the site energy of a site with coordination number z, as ϵ_z. Therefore, in an Nth generation lattice ϵ_i's range from ϵ_2 to $\epsilon_{z_{max}}$, where $z_{max}=2^{N-1}$.

Let us, first of all, try to give a simple intuitive picture of how to construct a wave function that will have a nontrivial disribution of amplitude over the entire lattice, irrespective of its size. We work with the difference equation version of the Schrödinger equation,

$$(E-\epsilon_i)\psi_i=t\sum_{j\in(NN\ of\ i)}\psi_j, \tag{2}$$

where, ψ_j is the amplitude of the wave function at the jth site. We shall be looking for energy eigenvalues that solve the above equation consistently all over the lattice and yet yield a nontrivial distribution of ψ_j's. We begin with $N=3$, i.e., where we have at least one class of sites with $z>2$ (here, z_{max} is 4). By inspection we can see that if we choose $E=\epsilon_2$, then this equation satisfies the Schrödinger equation consistently at all sites if we demand that the amplitude of wave function vanishes at sites with coordination number four. Extending this idea, we find that indeed we can satisfy Schrödinger equation locally at all sites in any arbitrarily large lattice provided we set the amplitude ψ_i at a site with coordination number z, equal to zero for all $z>2$. That is, in this scheme, the electron cannot ''feel'' the presence of the sites with $z>2$. This helps in attaining an ''extended'' character of the wave function. It may be mentioned here that, a similar technique was used earlier in the case of a Vicsek fractal.[9] Here, it is to be noted that the solutions of the Schrödinger equation on such a lattice will be highly degenerate and we are exhibiting one such case only. In Figs. 2(a) and 2(b), the amplitude distributions for $E=\epsilon_2=0$ are given on a lattice with $N=3$ and $N=4$, respectively. The pattern of distribution of the amplitudes for the $N=4$ case has been obtained by joining the extremeties A and B [Fig. 2(a)] of the basic $N=3$ plaquette (which now becomes the building block for the next generation) side by side such that the vertex B of one plaquette falls on the vertex A of the next [Fig. 2(b)]. Following this strategy we can use the $N=4$ plaquette as the basic unit and determine the amplitude-distribution on an $N=5$ lattice by joining the $N=4$ plaquettes at the vertices having $z=8$, where the amplitude of the wave function is zero. The method can be extended easily to construct the distribution pattern for higher generations.

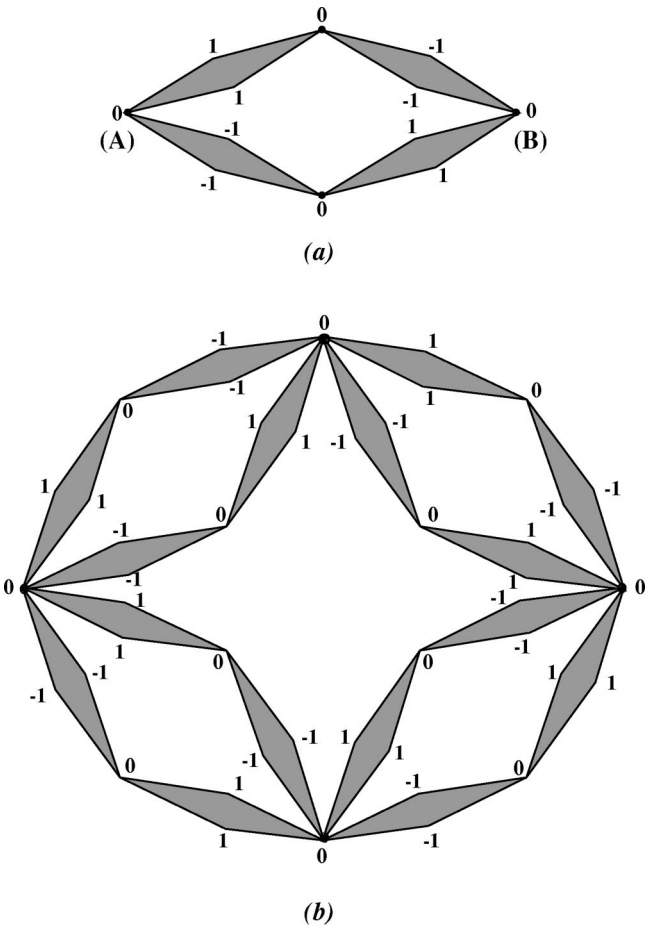

FIG. 2. Distribution of the amplitudes of an extended wave function at $E=0$ on (a) an $N=3$ lattice and (b) an $N=4$ lattice. All ϵ_i's have been set equal to zero and $t=1$. ψ_i's take on values -1, 0, and 1 on different sites (i).

The above idea can now be coupled to an RSRG scheme to extract other possible energy eigenvalues. Starting from any arbitrarily large finite version of the hierarchical diamond lattice, we can renormalize the lattice n times. The recursion relations for the site energies and hopping integral are, respectively, given by

$$\epsilon_z(n)=\epsilon_{2z}(n-1)+\frac{2zt^2(n-1)}{[E-\epsilon_2(n-1)]} \tag{3}$$

and

$$t(n)=\frac{2t^2(n-1)}{[E-\epsilon_2(n-1)]}. \tag{4}$$

Here, $\epsilon_z(n)$ and $t(n)$ denote, respectively, the values of the on-site potential and the hopping integral, at the nth stage of renormalization.

Now, suppose we start with an Nth generation lattice. We renormalize this lattice $n=N-3$ times and bring it down to an ''effectively'' third generation lattice where $z=2$ and 4. The site energies and the hopping integral for this renormalized version are calculated using Eqs. (3) and (4). We then apply the earlier trick on this renormalized lattice, i.e., we make the amplitudes ψ_i vanish at the $z=4$ sites (on the rescaled version). This happens for $E=\epsilon_2(n)$. This is a polynomial equation in E. Though, it is difficult to provide a complete proof, we have performed explicit calculations up to $N=5$ and for $n=1$ and 2. In each case it turns out that the solutions of the equation $E=\epsilon_2(n)$ satisfy the Schrödinger equation all over the lattice with a nontrivial distribution of the amplitudes of the wave function. Therefore, it is tempting to make the conjecture that the real roots of the equation $E=\epsilon_2(n)$ will correspond to the ''extended'' eigenstates in the general case, if we follow the same prescription. When mapped onto the original lattice the wave function will vanish at sites with some value of z onwards and will remain finite at all other lattice points with lower values of z. Let us clarify this idea by discussing two specific situations. First, we choose $E=\epsilon_2(1)$. Now, we should have at least an $N=4$ lattice as our starting structure, where $z_{max}=8$. We then renormalize it once to cast it into an effective $N=3$-stage lattice with $z_{max}=4$. We find that the solutions of the equation $E=\epsilon_2(1)$ are $E=\pm 2$, with all $\epsilon_z=0$ initially and $t=1$. Each of these energy values consistently satisfies the Schrödinger equation everywhere on the renormalized lattice provided we fix $\psi_i=0$ at the $z=4$ sites on this renormalized structure. When mapped onto the original $N=4$-stage hierarchical structure, we see that the amplitude of the wave function vanishes only at the vertex with the highest coordination number, i.e., 8 and it is nonzero at all other points. The rule for constructing such a distribution may be formulated as follows. We consider two plaquettes, type I and type II, each being an $N=3$ diamond lattice. In Fig. 3(a), the plaquette I is shown. In type II, each nonzero amplitude of the wave function is of opposite sign to that at the corresponding vertex in type I. In both of them, the Schrödinger equation is satisfied at each vertex for $E=2$. These two plaquettes, I and II, are then joined end to end in an alternate fashion such that the vertices with $\psi_i=0$ fall on each other [Fig. 3(b)]. In this way, the $N=4$ structure is built, and by selecting the $N=4$ structure as the basic unit we can construct the pattern for an $N=5$ lattice by joining them suitably at the extreme points (with $\psi_i=0$). The scheme can go on for other higher order lattices. It is to be appreciated that, for higher generations ψ_i will remain zero at all vertices with $z=8,16,32,\ldots$. The even number of the nearest-neighbors helps maintaining the value of ψ_i equal to zero at these sites. For the other sites with $z<4$ at any generation, the Schrödinger equation is locally satisfied everywhere, once it is satisfied at a lower generation.

Second, we consider $E=\epsilon_2(2)$. Here, we need to start from an $N=5$ structure. We map this lattice onto an $N=3$ version with renormalized parameters and force $\psi_i=0$, as before, at the sites with $z=4$ on this renormalized lattice. The effective $N=3$ lattice with the distribution of the amplitudes is presented in Fig. 4(a). When unfolded to retrieve the original $N=5$ lattice, we now find that the amplitude is zero only at the vertices with $z=16$, and three new amplitudes $\pm a$, $\pm b$, and $\pm c$ appear at $z=2$ and 4 sites. One quarter of the full $N=5$ lattice is shown in Fig. 4(b) with the distribution of the values a, b, and c. The complementary portion with $-a$, $-b$, and $-c$ is not shown, but can easily be conceived of. In order to satisfy the Schrödinger equation consistently at each vertex, we see that a, b, and c should take values $(E^2-8)/8E$, $(E^2-8)/8$, and $E/8$, respectively, pro-

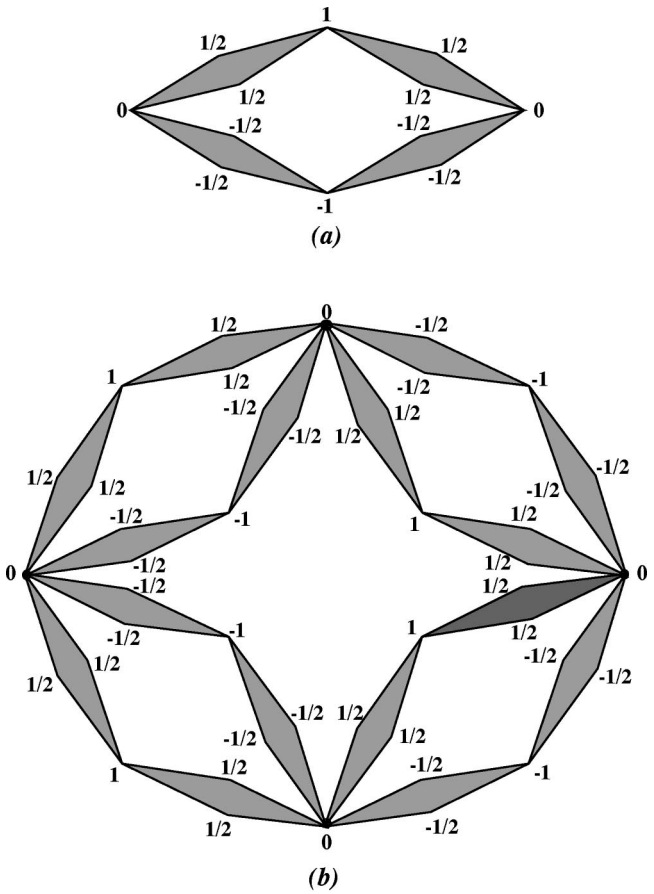

FIG. 3. Amplitudes of a wave function on (a) the basic plaquette *I*, which acts as a building block of an $N=4$ lattice and (b) an $N=4$ lattice for $E=\epsilon_2(1)$. All ϵ_i's have been set equal to zero and $t=1$. ψ_i's take on values -1, $-1/2$, 0, 1/2, and 1 on different sites (i).

vided the energy E is a solution of the equation $E^4-12E^2+16=0$. This is precisely the polynomial equation that is obtained by setting $E=\epsilon_2(2)$. We thus confirm that for every root of this equation we are able to construct extended eigenstates consistent with the Schrödinger equation. We expect that, reasoning in the same manner as in the above cases, we are likely to uncover a whole set of eigenvalues by solving the equation $E=\epsilon_2(n)$ with $n=1,2,3,\ldots$, in an arbitrarily large lattice for which the wave function will be ''extended'' in the sense described earlier.

Before ending this section, we point out two other possibilities of getting ''extended'' type of states:

(i) We consider the third generation lattice, where $z_{max}=4$. It is quite obvious that by choosing $E=\epsilon_4$ (which happens to be equal to ϵ_2 in the bare length scale in our model), we can have a consistent solution of the Schrödinger equation on this lattice in which the amplitude of the wave function is zero on all vertices with $z=2$ and alternates between ± 1 on the vertices with $z=4$. This is, of course, one of the possible configurations. But we can carry on the process of construction for other eigenvalues by setting $E=\epsilon_4(n)$ for an $N=n+3$ generation lattice, and by demanding that ψ_i vanishes identically on all $z=2$ vertices on the n step-renormalized version of the same lattice. In Fig. 5, we exhibit the distribution of amplitudes for $E=\epsilon_4(1)$ on a fourth generation lattice. Distribution for higher generations and

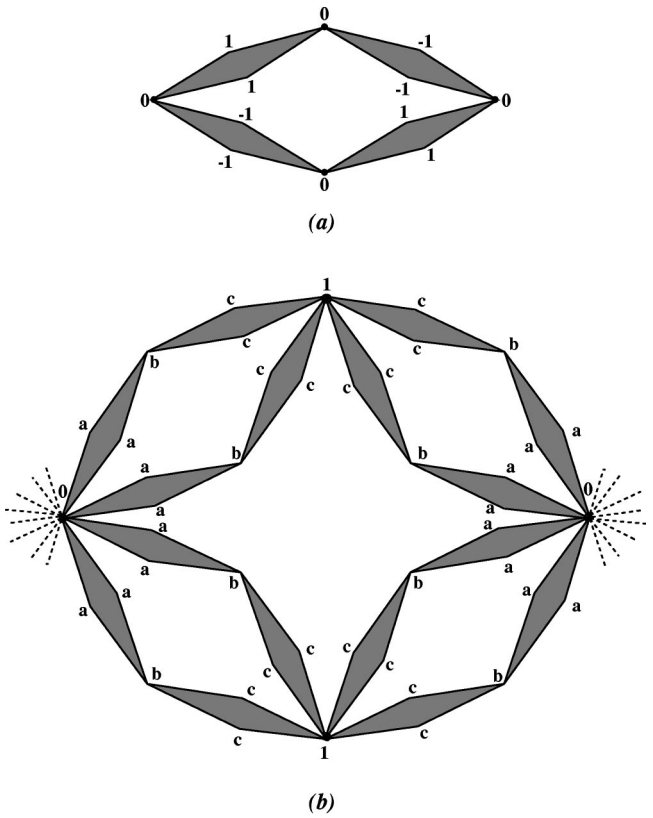

FIG. 4. Amplitude distribution on (a) an effectively $N=3$ plaquette obtained by renormalizing an $N=5$ lattice twice and (b) one quarter of the original $N=5$ version for $E=\epsilon_2(2)$. Dashed lines at the two extreme vertices indicate the presence of complementary plaquettes. All ϵ_i's have been set equal to zero and $t=1$. The values of *a*, *b*, and *c* are given in the text.

other eigenvalues can be obtained by extending the earlier ideas. Similar results can be obtained for the general case with $E=\epsilon_{2z}(n)$.

(ii) The other possibility refers to a specific initial choice of the site energies. By looking at the recursion relations (3)

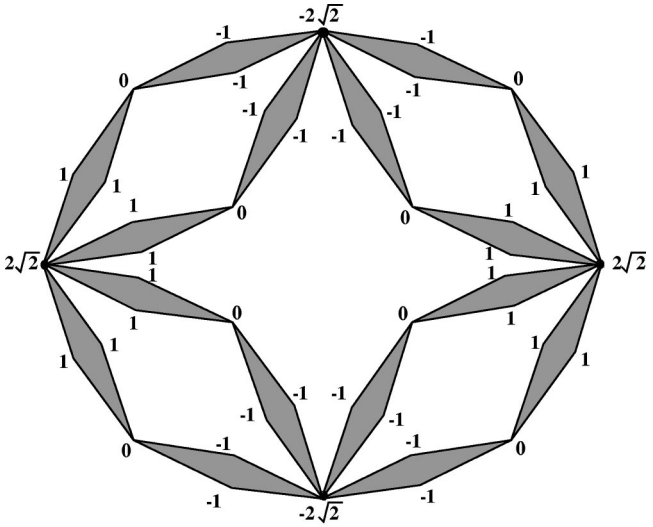

FIG. 5. Amplitudes of a wave function for $E=\epsilon_4(1)$ on an $N=4$ lattice. All ϵ_i's have been set equal to zero and $t=1$. ψ_i's take on values $-2\sqrt{2}$, -1, 0, 1, and $2\sqrt{2}$ on different sites (i).

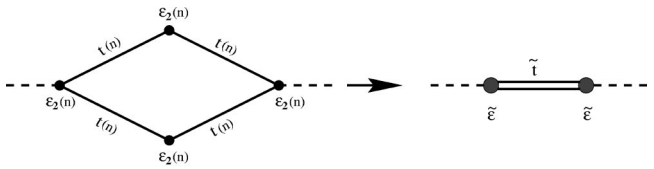

FIG. 6. Reduction of an n times renormalized lattice to an effective dimer. The leads are shown as dashed lines.

and (4) for the on-site term and hopping integral, respectively, we find that if we start with a model where $\epsilon_{2z}=\epsilon_z - zt$, then for $E=\epsilon_2+2t$, each on-site term and the hopping integral exhibit a fixed point behavior. It implies that the nearest neighbor hopping integral does not flow to zero under iteration and we have a nonvanishing ''connection'' between nearest-neighboring sites at all length scales. This is a clear signature of the corresponding eigenstate being extended.[14]

We will now describe how to investigate the transmission characteristics of a diamond hierarchical lattice. We will emphasize on the analytical treatment of the recursion relations and will discuss the behavior of the transmission coefficient $T(E)$ for the two cases with $E=\epsilon_2$ and $E=\epsilon_2(1)$, respectively, for which the transmission coefficient displays totally opposite characteristics. $T(E)$ for the other energies $[E=\epsilon_{2z}(n)]$ can be obtained by following the method adopted for these cases.

III. ANALYSIS OF THE TRANSMISSION COEFFICIENT

For calculating the transmission coefficient $T(E)$, we attach two semi-infinite perfectly ordered leads to the two ''diametrically'' opposite vertices having the maximum coordination number in any generation N. The original lattice is then renormalized $n(=N-2)$ times, so that we are left with a basic $N=2$ rhombus with four vertices each having an effective site energy $\epsilon_2(n)$ and the nearest-neighbor hopping integral $t(n)$ [Fig. 6]. This elementary rhombus is now folded into a ''dimer'' with site energy and nearest-neighbor hopping integral, respectively, given by

$$\tilde{\epsilon}=\epsilon_2(n)+\frac{2t^2(n)}{[E-\epsilon_2(n)]}$$

and

$$\tilde{t}=\frac{2t^2(n)}{[E-\epsilon_2(n)]}.$$

Following the standard procedure[15] it is then easy to show that

$$T(E)=\frac{4\sin^2 k}{[P_{21}-P_{12}+(P_{22}-P_{11})\cos k]^2+(P_{11}+P_{22})^2\sin^2 k}, \tag{5}$$

where the elements of the matrix P are

$$P_{11}=\left[\left(\frac{E-\tilde{\epsilon}}{\tilde{t}}\right)^2-1\right]\frac{\tilde{t}}{t_0}, \tag{6}$$

$$P_{12}=-\left(\frac{E-\tilde{\epsilon}}{\tilde{t}}\right)=-P_{21}, \tag{7}$$

and

$$P_{22}=-\frac{t_0}{\tilde{t}}. \tag{8}$$

Here, $k=\cos^{-1}[(E-\epsilon_0)/2t_0]$, where, ϵ_0 and t_0 refer to the on-site potential and the hopping integral, respectively, of the ordered lead.

In order to understand the behavior of $T(E)$ for large systems at any particular energy, we must analyze how the matrix elements P_{ij} behave for large number of RG iterations, n. As a lattice of any generation should finally be reduced to a basic $N=2$ rhombus having only $\epsilon_2(n)$ and $t(n)$ (see Fig. 5), we must analyze the flow patterns of ϵ_2 and t under successive RSRG iterations. The evolution of these two parameters ultimately controls $\tilde{\epsilon}$ and $\tilde{t}$, and hence the matrix elements P_{ij}. Let us discuss it for two specific cases. Throughout the analysis, we will set all $\epsilon_z=0$ and $t=1$.

Case (i) $E=\epsilon_2=0$

From direct calculations we find that as $E\to 0$, the leading behaviors (in E) of t and ϵ_2 are given by,

$$t(n)\sim\frac{2t^2}{E} \tag{9}$$

$$\epsilon_2(n)\sim\frac{-4t^2}{E} \tag{10}$$

for $n=2$, 3, 4, and 5. We thus assume these forms to be true for any arbitrary value of n, viz., $n=m$. Then proceeding according to the standard method of induction, we can prove, using the recursion relations (3) and (4) that Eqs. (9) and (10) indeed hold good for $n=m+1$ as well. Therefore, we accept the above forms as the leading terms in the expressions for $t(n)$ and $\epsilon_2(n)$ for $E\to 0$ and for any arbitrary value of n with $n\geqslant 2$. It is now easy to work out an expression for $(E-\tilde{\epsilon})/\tilde{t}$, which is given, to the leading order in E, by

$$\frac{E-\tilde{\epsilon}}{\tilde{t}}=\frac{E^2}{2t^2}+1. \tag{11}$$

A direct subtitution of the above result in the expressions of P_{ij} shows that, $P_{11}=P_{22}\to 0$ and $P_{12}=-P_{21}=-1$ as $E\to 0$. The expression for $T(E=0)$ now becomes

$$T(E=0)=\frac{4\sin^2 k}{[P_{21}-P_{12}]^2}=\sin^2 k=1-\frac{\epsilon_0^2}{4t_0^2}. \tag{12}$$

If we select $\epsilon_0=0$, then $T(E=0)$ is unity, and any arbitrarily large diamond hierarchical lattice becomes completely transparent to an incoming electron with $E=0$.

Case (ii) $E=\epsilon_2(1)=2$

The central idea of the analysis for case (i) can now easily be extended to study the general situation, where $E=\epsilon_2(n)$.

The results, however, turn out to be totally different, as we have checked numerically by solving the equation $E-\epsilon_2(n)=0$ for several values of n. We present below analytical results for $E=\epsilon_2(1)=2$ for steps $n\geqslant 2$. Once again, we observe the behavior of t and ϵ_2 around $E=2$ for successive iterations. We set $E=2+\delta$, δ being infinitesimally small, and find, by direct calculation that, for $n\geqslant 2$

$$\epsilon_2(n)=\frac{2}{\delta}+f_n+\mathcal{O}(\delta) \tag{13}$$

$$t(n)=-\frac{1}{\delta}+g_n+\mathcal{O}(\delta) \tag{14}$$

up to leading order in δ, where f_n and g_n are, respectively, given by

$$f_n=\frac{2^{2n}}{9}+\frac{2n}{3}-\frac{11}{18}$$

and

$$g_n=\frac{2^{2n-2}}{9}-\frac{n}{3}+\frac{35}{36}.$$

The above forms set in, as in case (i), after the first iteration, and hold perfectly well for $n=2, 3, 4$, and 5. We now make use of the recursion relations (3) and (4) to find that the result is true for $(n+1)$th stage as well. Thus, we take Eqs. (13) and (14) to represent the general n behavior of $\epsilon_2(n)$ and $t(n)$ for any $n\geqslant 2$. The leading δ behavior of $\tilde{\epsilon}$ and $\tilde{t}$ are obtained to be,

$$\tilde{\epsilon}=\frac{1}{\delta}+\frac{f_{n+1}}{2}$$

and

$$\tilde{t}=-\frac{1}{\delta}+g_{n+1}$$

respectively, which lead to the equation

$$\frac{E-\tilde{\epsilon}}{\tilde{t}}=\frac{2^{2n}}{3}-\frac{4}{3}. \tag{15}$$

However, it should be noted that in order that the present analysis is valid, we must have a finite (although large) number of iterations n such that f_n and g_n are also finite quantities and are small compared to $1/\delta$ as $\delta\to 0$. The matrix elements now read (neglecting terms of the order of δ^2 and taking the limit $\delta\to 0$), $P_{11}=-2(4^n-4)/3$, and $P_{22}=0$, $P_{12}=-P_{21}=-1$. For large (but finite) values of n one can now show, using Eq. (5) that

$$T_n(E=2)\sim 2^{-4n}.$$

It is quite obvious that, depending on the value of the energy, one has to select the on-site term and the hopping integral for the lead suitably, so that the energy does not fall beyond the ''band'' of the ordered lead or even coincide with the ''band edge.'' In both cases the use of the expression (5) will not be meaningful.

We have also numerically calculated $T(E)$ for $E=\epsilon_2(2)$, $E=\epsilon_2(3)$, and $E=\epsilon_4(1)$ for lattices starting from $N=3$ up to $N=6$. We observe a gradual attenuation in the value of the transmission coefficient according to our expectation. The present results thus provide an example of what we may call an ''atypical'' extended state[9] where, though the wave function displays nonzero amplitudes even at the farthest portions of an arbitrarily large lattice, the end-to-end transmission decays with increasing lattice size. We expect similar behavior of $T(E)$ for other cases [with $E=\epsilon_{2z}(1)$] also.

The fixed point behavior of $T(E)$

Before ending this section, we discuss the case where we have a fixed-point behavior of the Hamiltonian parameters. For this, we take a model with $\epsilon_{2z}=\epsilon_z-zt$ and set $E=\epsilon_2+2t$. All parameters then remain unaltered under RSRG and we find that for $\epsilon_2=0$ and $t=1$, the numerical value of the transmission coefficient is given by,

$$T(t_0)=\frac{4(t_0^2-1)}{t_0^4}, \tag{16}$$

where we have set the site energy of the lead ϵ_0, equal to zero. Naturally, we have to choose any suitable value for t_0 so that the above energy remains within the ''band'' of the ordered lead. The above expression for T remains fixed for arbitrarily large versions of a diamond lattice. The wave function is definitely extended as t does not flow to zero under RSRG.

IV. CONCLUSION

We have presented a hierarchical lattice model where the coordination number of the lattice points range from 2 to 2^{N-1} depending on the generation index N. In such a lattice there exist ''extended'' type of electronic states some of which have been identified and the corresponding eigenvalues have been calculated using renormalization-group ideas. We also presented an exact analysis of the end-to-end transmission coefficient to reveal that the lattice, irrespective of its size, becomes completely transparent to an electron with energy $E=0$, while for other energies the transmission coefficient has a scaling behavior. We obtained an exact form of the scaling for a specific energy, and the other forms can be obtained using the same methodology, though we did not present the other analytical results here.

*Electronic address: papluchakrabarti@hotmail.com
†Electronic address: bibhas@cmp.saha.ernet.in
‡Electronic address: arunava@klyuniv.ernet.in; rkm@cmp.saha.ernet.in

[1] R.B. Griffiths and M. Kaufman, Phys. Rev. B **26**, 5022 (1982).
[2] A.N. Berker and S. Ostlund, J. Phys. C **45**, 4961 (1979).
[3] Y. Gefen, B. Mandelbrot, and A. Aharony, Phys. Rev. Lett. **45**, 855 (1980).
[4] S. Alexander and R. Orbach, J. Phys. (France) Lett. **43**, L625 (1982); J.R. Banavar and M. Cieplak, Phys. Rev. B **28**, 3813

(1983).
[5] E. Domany, S. Alexander, D. Bensimon and L.P. Kadanoff, Phys. Rev. B **48**, 3110 (1983).
[6] R. Rammal and G. Toulouse, Phys. Rev. B **49**, 1194 (1982).
[7] W. Schwalm and M. Schwalm, Phys. Rev. B **39**, 12 872 (1989); **47**, 7847 (1993).
[8] X.R. Wang, Phys. Rev. B **51**, 9310 (1995); A. Chakrabarti, J. Phys.: Condens. Matter **8**, 10 951 (1996).
[9] A. Chakrabarti and B. Bhattacharyya, Phys. Rev. B **54**, R12 625 (1996); A. Chakrabarti, J. Phys.: Condens. Matter **8**, L99 (1996).
[10] D.H. Dunlap, H.-L. Wu, and P. Phillips, Phys. Rev. Lett. **65**, 88 (1990).
[11] A. Chakrabarti, S.N. Karmakar, and R.K. Moitra, Phys. Rev. B **50**, 13 276 (1994).
[12] A.N. Samukhin, V.N. Prigodin, and L. Jastrabik, Phys. Rev. Lett. **78**, 326 (1997).
[13] C.P. Zhu, S.J. Xiong, and T. Chen, Phys. Rev. B **58**, 12 848 (1998).
[14] B.W. Southern, A.A. Kumar, P.D. Loly, and A.-M.S. Tremblay, Phys. Rev. B **27**, 1405 (1983).
[15] A. Douglas Stone, J.D. Joannopoulos, and D.J. Chadi, Phys. Rev. B **24**, 5583 (1981).