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MARKET APPLICATION OF THE PERCOLATION MODEL: RELATIVE PRICE DISTRIBUTION

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We study a variant of the Cont–Bouchaud model, which utilizes the percolation approach of multi-agent simulations of the stock market fluctuations. Here, instead of considering the relative price change as the difference of the total demand and total supply, we consider the relative price change to be proportional to the "relative" difference of demand and supply (the ratio of the difference in total demand and total supply to the sum of the total demand and total supply). We then study the probability distribution of the price changes.

Keywords: Econophysics; Monte Carlo; simulation; Cont–Bouchaud model.

1. Introduction

Statistical physics contains the methods for extracting the average properties of a macroscopic system (matter in bulk) from the microscopic dynamics of the systems. It also gives us precise knowledge of the fluctuations (above these averages) of these quantities.[1] Scaling laws, experimental or theoretical, have been of special interests to physicists. Hence, physicists are trying to employ these methods to study the fluctuations of the stock markets (the study of which began with the work of Louis Bachelier in 1900[2]) as well. With access to large sets of data from financial markets, an extensive search for such scaling laws has begun recently.[3] Fluctuations over the average, say in some stock prices, are of immense interest to the economists also. The nature of these fluctuations, whether random or otherwise, are of extreme importance. Stigler[4] studied the market fluctuations by employing Monte Carlo methods more than thirty years back. The fluctuations are believed to follow a Gaussian distribution for long time intervals. Mandelbrot[5] was first to observe a clear departure from Gaussian behavior for these fluctuations for short time intervals. There have been various explanations and descriptions for it, ranging from power laws, exponentials to multi-fractal behavior.

The percolation[6] approach of Cont and Bouchaud[7] is one of the simplest of the numerous multi-agent simulations of the stock market fluctuations. Monte Carlo simulations of the above model made at the percolation threshold, show power-law "fat" tails for short time intervals and exponential truncation for longer time intervals. The model is also consistent with the weak correlations between successive changes of price and strong correlations between successive values of changes of price. There has been various variants of the Cont–Bouchaud model.[8] Here, we study another variant of the Cont–Bouchaud model where instead of considering the relative price change coming from the difference of the total demand and total supply, we consider the relative price change to be proportional to the "relative" difference of demand and supply (i.e., the ratio of the difference in demand and supply to the sum of demand and supply).

2. The Model and Results

We human beings are "social animals" and hence like to stay together and are influenced by others at all spheres of life. At most occasions, we form "clusters", which for simplicity, will be considered as random. As in the case of percolation theory of random graphs, the traders are assumed to just form random clusters and share their opinions. The history of the price changes and the limitations in the disposable capital of each trader are ignored.

The original Cont–Bouchaud model[7] considered the mean-field limit of infinite-range interactions instead of the usual nearest-neighbor percolation on lattices.[6] In the variant models,[8] the sites of a d-dimensional lattice are randomly occupied with probability p and empty with probability $(1 - p)$ and the occupied nearest-neighbors form clusters. Each cluster containing N_t traders decides randomly, to buy (with probability a), sell (also with the same probability a), or to remain inactive (with probability $1 - 2a$). So far, the relative change of the price was considered to be proportional to the difference between the total demand and the total supply. Hence, for any time step Δt, we first find the existing clusters and the number n_s of clusters, each containing s traders. Then each cluster randomly decides whether to buy, sell or remain inactive with the above mentioned probabilities. The parameter a is called the "activity" and the increase in activity is equivalent to the increase in the time unit, since a is the fraction of traders, which are active per unit time. Thus small a correspond to small time intervals and large a (with the maximum of 0.5) correspond to large time intervals. Then, the relative price change for one time step is considered proportional to the difference of the total demand and total supply:

$$R(t) = \ln P(t + \Delta t) - \ln P(t) \propto \sum_s n_s^{\text{buy}} s - \sum_s n_s^{\text{sell}} s \,, \tag{1}$$

where the constant of proportionality is taken to be unity.

If we take one time step Δt to be very small so that only one cluster of traders can trade during this time interval (the number of clusters trading in one time step

$N = aN_t \sim 1$), then the probability distribution $P(R)$ is completely symmetric about zero (as in real stock markets) and just follows the distribution n_s of clusters. The distribution, right at $p = p_c$ is $n_s \propto 1/s^\tau$ with $2 < \tau < 2.5$ in two to infinite dimensions.[6] If the time step Δt is large so that all the traders can trade in each time step ($N \sim N_t$), then the probability distribution $P(R)$ is closer to a Gaussian. When the time step is in the intermediate range so that $1 \ll N \ll N_t$ the price changes are bell-shaped with power-law tails. This crossover to Gaussian behavior with the variation of a is observed in reality also.[9]

In our model, the relative price change is proportional to the "relative" difference of demand and supply, i.e., the ratio of the difference in demand and supply, and the grand total of demand and supply:

$$R(t) = \ln P(t + \Delta t) - \ln P(t) \propto \frac{\sum_s n_s^{\text{buy}} s - \sum_s n_s^{\text{sell}} s}{\sum_s n_s^{\text{buy}} s + \sum_s n_s^{\text{sell}} s}\,, \tag{2}$$

where the constant of proportionality is again taken to be unity.

Our computer simulations first distribute sites randomly on the square lattice of dimensions $L \times L$ at the percolation threshold ($p = p_c = 0.592746$) and then determine the clusters. For each time step Δt, we allow each cluster to decide randomly whether to trade or remain inactive. The trading clusters then again randomly decide to buy or sell, and then Eqs. (1) or (2) determine the relative

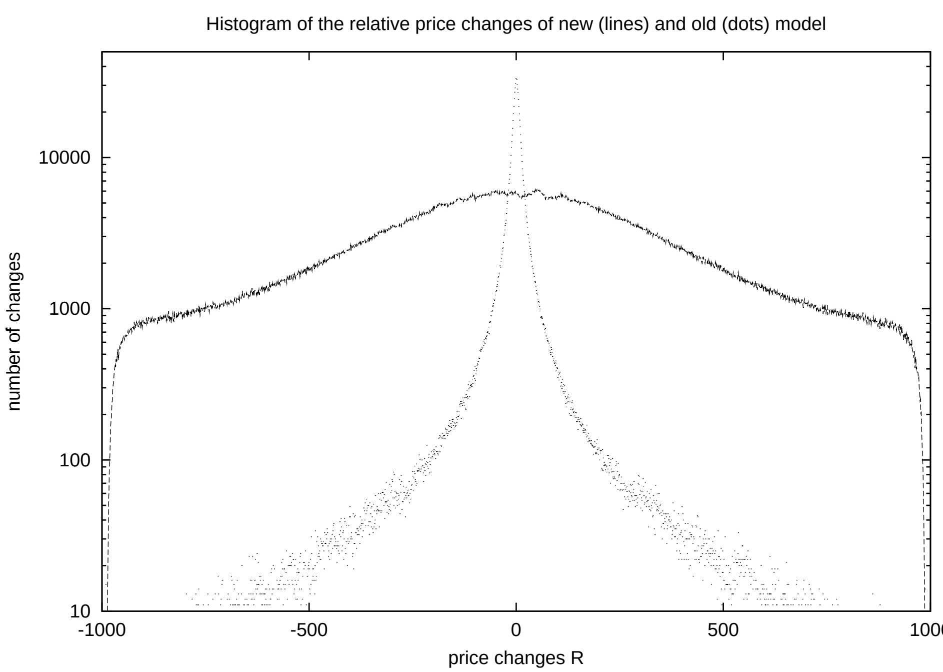

Fig. 1. Histogram of relative price changes plotted in the linear–logarithmic scale, obtained from computer simulations made at the percolation threshold for 5000 square lattices of size 1001×1001, 1000 time intervals and activity 0.01.

price change. We average over many lattice configurations to find the probability distribution $P(R)$. Programs in C and FORTRAN, based on the Hoshen–Kopelman algorithm in two dimensions (where we have considered site percolation with free boundary conditions) are available from the author.

The histograms of price changes, which we get when the relative price changes are determined according to Eq. (1) and those according to Eq. (2) are shown in Fig. 1, with $p_c = 0.592746$. In the Cont–Bouchaud model, we also see a crossover from a power-law to a bell-shaped behavior (within the accuracy of the computer simulations) for increase in activity a (not shown in the figure) showing its similarity with real stock markets. In this model, since the magnitude of the relative price change always lies between zero and unity, we observe a sharp cut-off in the histogram, unlike in real markets. Thus the original Cont–Bouchaud model is superior to this model, in this respect.

3. Discussions and Summary

We study a variant of the Cont–Bouchaud model: the relative price changes are defined as the ratio of the difference in demand and supply to the sum of demand and supply,

$$R(t) = \frac{\sum_s n_s^{\text{buy}} s - \sum_s n_s^{\text{sell}} s}{\sum_s n_s^{\text{buy}} s + \sum_s n_s^{\text{sell}} s},$$

where the constant of proportionality is taken to be unity. We also present some of the previous results of a variant of the Cont–Bouchaud model for comparison. We observe a sharp cut-off in the histogram for this model, unlike in real markets, which shows that the original Cont–Bouchaud model is superior to this model in this respect. This model too could be made more realistic, e.g., including the history of the price changes.

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