Trade and Wage Inequality: A Specific Factor Model with Intermediate Goods

Alokesh BARUA and Manoj PANT

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Abstract

In this paper we have made an attempt to explain the observed rising inequality between unskilled and skilled wages, or, fall in relative wages of unskilled labour within a general equilibrium framework by introducing specific factors and non-traded intermediate goods. For this we set up two distinct models of trade. The first model shows that for a small economy with specific factors freer trade may cause (a) both skilled and unskilled wages to rise, and, (b) the two wage rates may move in opposite directions depending on the trade – induced patterns of specialization. The model also suggests that while trade may increase wage inequality, this does not imply that poverty increases as wages of unskilled workers increase. By extending the model to incorporate intermediate goods we have shown that wage inequality may unambiguously increase (decrease). However, it is seen that this is a consequence of the structure of trade and not trade per se.

JEL Listing: F11,F13
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Introduction:

There has been a proliferation of studies in recent years purporting to examine the effects of international trade on relative wages, that is, the real wages of unskilled labour relative to that of skilled labour. In contention is the prediction of the standard Heckscher – Ohlin – Samuelson (HOS) model that trade must lead to an increase in real wages paid to labour relative to a second factor, normally capital, at least for countries with abundant labour. Thus, in a two factor version of this model with unskilled and skilled labour as the two relevant factors (Leamer, 1993, 1995), it tries to explain why freer trade might cause a fall in the relative wages in the United States. Following the usual Stolper – Samuelson (SS theorem) arguments it suggests that the US being a relatively skilled – labour abundant economy exports relatively skill intensive goods as compared to its imports which are unskilled- labour intensive. Extending this to a three factor model by assuming that capital is the third factor and further assuming (as seems reasonable) that capital and skilled labour are complimentary, then an application of the HOS model would imply, in contrast, a rise in relative real wages paid to unskilled labour in developing countries. Therefore, there is enough flexibility in the model to incorporate any real world phenomenon that one may encounter with. In other words, paraphrasing the relative wage hypothesis in terms of wage inequality, we may say that a rise (or fall) in relative real wages (as defined above) is synonymous with a fall (or rise) in wage inequality. Simply, an increase in unskilled real wage (meaning, a rise in relative real wage) essentially implies a fall in wage inequality and vice versa.

The HOS model, of course, only applies to trade between countries which differ somewhat in their endowments as, for example, in the case of trade between developed (N) and developing countries (S). Hence, the model also implies that as wage inequality declines in one country, it must increase in the other. However, empirical studies, by and large, do not support this hypothesis and show that increased trade is in fact accompanied by increasing wage
inequality (a decline in relative real wages) between unskilled and skilled labour in both developing and developed countries (Cline, 1997; for an extensive survey see Anne Harrison et.al, 2010).

As against the relative wage hypothesis, an absolute wage hypothesis is defined as a situation where real wages of both unskilled and skilled workers may either rise or fall (See, Beladi and Batra, 2004). It is nonetheless possible to construe a relative wage hypothesis within an absolute wage hypothesis if the fall (or rise) in unskilled wage is at a higher (lower) rate than the fall (or rise) in the skilled wage rate. For instance, while real wages of most workers fell between 1973 and 1993 in the US (Cline, 1997), the skill labour wage fell at a much slower rate than the unskilled wage; however, both skilled and unskilled wages were rising since 1995 (Beladi and Batra, 2004). The existing theoretical literature only tries to explain relative wage hypothesis where both skilled and unskilled wages move in opposite direction, and, therefore, it contradicts the absolute wage version (Beladi and Batra, 2004), though a relative wage hypothesis as mentioned above can be established within an absolute wage hypothesis.

Batra and Slottje (1993) and Batra and Beladi (2004) tried to model this issue of change in relative and absolute wages. In Batra and Slottje (1993), a tricky argument has been put forward that a fall (rise) in the average wage implies a decline (rise) in unskilled wage rate, since unskilled workers are assumed to earn a negative premium over the average wage, w, which is determined competitively in the market. However, unskilled labour is not modelled specifically and the assumption is that a fall in average wage implies a decline in unskilled wages. This must assume that the distribution of unskilled and skilled workers remains unchanged. Without this assumption, the average wage could change even with relative wages of skilled and unskilled workers remains unchanged. We have corrected for this in our models.

In general, theoretical literature on this subject is scanty. Earlier work has tried to reconcile the empirical results with the HOS theory in one of two ways. One, it is shown by arguing that the issue is unrelated to trade; and wage inequality is increasing because of technology bias against unskilled labour (see, Adrian Wood, 1997). Two, that some of the assumptions of the HOS theorem are violated. Thus, (Deardorff, 2001; Xiang, 2007) have argued that in presence of multiple cones of diversification between countries, increased wage inequality
may arise as a theoretical possibility. In one paper, (Beladi and Batra, op.cit.) tried to address this issue by incorporating non–traded goods and specific factors in a general equilibrium model of trade. If unskilled labour is specific to the non-traded goods sector, wage inequality could increase if the output of the non-traded goods falls with increased trade and wages of unskilled labour fall even faster than of skilled workers.

More recent models have looked at the issue of trade and wage inequality in the context of models where production is outsourced. Here, while Batra and Beladi (2010) have introduced outsourcing in a general equilibrium HOS model, others have departed from the perfectly competitive general equilibrium models by arguing that products are heterogeneous and consist of a number of processes some being skilled labour intensive and others using more of unskilled labour. Over time, as transport costs decline the production is fragmented with unskilled labour intensive processes outsourced to developing countries. In the latter, however, these processes are relatively skilled labour intensive. Hence, wage inequality will tend to increase in both sets of countries after trade. This is obviously a departure from the HOS assumptions of product homogeneity and non-factor intensity reversal. There are various variants of this basic approach. For an extensive survey see Harrison, et.al. (op.cit)

In this paper we have made an attempt to explain the phenomenon of increasing relative wage inequality in an HOS general equilibrium framework using two separate models. In the first, we introduce a specific factor, and, in the second, we include a non-traded intermediate good used as an input in the production of the other final goods, It is then shown that increased trade can lead to increased or decreased wage inequality even for a small developing country. We have also shown that the increasing relative wage inequality can co-exist with an increase in the absolute wages of both skilled and unskilled labour. The conditions under which this happens are also fairly simple unlike in earlier models on this issue. By looking at absolute wages of unskilled labour we are also able to relate our arguments to the discussion on the link between trade and poverty which have assumed serious political overtones in both developing and developed economies. Thus, we are able to show that even if trade leads to an increase in wage inequality that does not necessarily imply that it also leads to an increase in poverty. In general, one does not need to take recourse to models of imperfect competition and/or outsourcing to
reconcile trade theory with the empirical evidence on changes in relative and absolute wages of skilled and unskilled wages.

In the next two sections we develop the two models. The model of Section 1 is a simple one with one specific factor while Section 2 extends the model to a non-traded intermediate good. Finally in Section 3 we conclude with some policy suggestions.

Section 1: Model 1

Assume there are two sectors $X_1$ (manufacturing) and $X_2$ (services) using capital ($K$) and labour ($L$). Capital, $K$, is specific to $X_1$ and unskilled labour, $L_u$, to $X_2$. Relative price of $X_2$ is $P$. In developing countries, the service sector does not fit the classic definition of services and generally includes a lot of self employed in the trade and hotel industries. Thus, for example, in India domestic wholesale and retail trade, real estate and construction services accounted for about 47 percent of the output of the services sector in 2007 (see, Banga and Kumar, 2009). Much of the labour in these industries is what we would classify as unskilled labour.

The production functions in the two sectors are given by

$$X_1 = X^1 (K_1, L_1)$$

$$X_2 = X^2 (L_u, L_2)$$

It is assumed both functions are linearly homogenous.

Assuming full employment of all inputs we have,

$$L_1 + L_2 = \bar{L}$$

$$L_u = \bar{L}_u$$

$$K_1 = K$$
The specifications in equations (4) and (5) reflect the assumptions that unskilled labour and capital are sector specific.

Given the factor market equilibrium conditions under competition, skilled wage, $W_s$, is equated in both sectors so that we have,

$$W_s = X_1^L = P X_2^L$$

(6) and (7)

Where $X_1^L$ and $X_2^L$ are the marginal physical productivities of skilled labour in sector 1 and 2 respectively.

Similarly, for unskilled labour we have

$$W_u = P X_2^U$$

(8)

The subscript, $u$, in above represents unskilled labor. Thus, $X_2^U$ stands for marginal physical productivity of unskilled labour in sector 2.

Therefore, the model has the following 8 endogenous variables, viz., $X_1$, $X_2$, $K_1$, $L_1$, $L_2$, $L_u$, $w_s$ and $w_u$ which are to be determined from the above 8 equations and the given three parameters, $\bar{K}$, $\bar{L}$ and $\bar{L}_u$. The model is thus internally consistent.

2.2. Comparative Statics

Results:

From (6) and (7) using (1), (2), (4) and (5) gives us

$$W_s = P X_2^L (L_u, L_2) = X_1^L (K, L_1)$$

(9)

\[\footnote{See Appendix A for the derivations of the results of this section.}\]
Equations (3) and (9) are two equations in two unknowns, \( L_1 \) and \( L_2 \). Totally differentiating the equations with respect to \( \bar{L}_U \) and \( \bar{L} \) we can show that

\[
\begin{align*}
\frac{dL_1}{d\bar{L}} &> 0 \\
\text{and} \\
\frac{dL_2}{d\bar{L}} &> 0
\end{align*}
\]

This does not depend on capital intensities since there is no substitution between \( L \), \( L_U \) and \( K \). Also, given specific factors, (9) implies from (1) and (2) that \( X_1 \) and \( X_2 \) must increase. This is the well known result that with specific factors the Rybczynski result breaks down (see, Bhagwati et.al.1998).

However, differentiating (3) and (9) totally with respect to \( \bar{L}_U \) gives us that

\[
\begin{align*}
\frac{dL_1}{d\bar{L}_U} &< 0 \\
\text{and} \\
\frac{dL_2}{d\bar{L}_U} &> 0
\end{align*}
\]

so that the increase in supply of the specific factor increases the output of the sector where it is employed.

The results for the Stolper - Samuelson (SS) effect are interesting. Thus, total differentiation of (9) gives

\[
dW^d/dP = p X^2_{LL} dL_2/dP + X^2_{L} = X^1_{LL} dL_1/dP
\]

From (3) and (12) we get
\[
\frac{dL_2}{dP} = - \frac{X^2_L}{(X^1_{1L} + p.X^2_{2L})} > 0
\]

and

\[
\frac{dL_1}{dP} < 0
\]

From (6) and (8) we get

\[
\frac{dW^S}{dP} = X^1_{1L} \frac{dL_1}{dP} > 0
\]

and

\[
\frac{dW^U}{dP} = X^2_{2L} + p.X^2_{2UL} \cdot \frac{dL_2}{dP} > 0
\]

where the signs in (14) are derived using (13).

Once again the standard SS result does not hold in the presence of specific factors since wages of both skilled and unskilled labour increase. The intuition is obvious. An increase in the price of \(X_2\) implies an increase in the value of the marginal product of both skilled and unskilled labour (by equations (6) – (8)) thus increasing wages all round. However, this implies a shift in skilled labour from \(X_1\) to \(X_2\) so that the production of \(X_2\) rises and that of \(X_1\) falls. Hence, an increase in the price of the product increases the return to both factors employed in its production.

Suppose this country exports commodity 2. Then, increased trade (an increase in \(P\)) would increase the wages paid to both skilled and unskilled labour as shown in (14). This result contradicts the SS results a la Leamer (1993, 1995) as discussed above. In other words, an increase in both skilled and unskilled wages with trade liberalization, as observed for both developed and developing countries, is perfectly consistent with a modified HOS model. Thus, we have the following proposition about the \textit{absolute wage hypothesis}:
Proposition 1

Trade liberalization for a small country where unskilled labour is specific to the export sector will lead to an increase in the absolute wage rates of both skilled and unskilled labour.

What about the issue of wage inequality? Here we get some interesting results.

Define, $W$, as the relative wage of unskilled to skilled labour, then

$$W = \frac{W^u}{W^s}$$

Then it can be shown using (12) and (14) that

$$\frac{dW}{dP} = \frac{dW^u}{dP} - \frac{dW^s}{dP} = X^2_U - X^2_L [1 + \lambda]$$

Where $\lambda = \frac{P \cdot X^2_{UL} - P \cdot X^2_{LL}}{X^1_{LL} + P \cdot X^2_{LL}}$ and $\lambda < 0$

Now, the absolute value of $\lambda$ being greater or less than unity determines the sign of $\frac{dW}{dP}$. More specifically,

$$\frac{dW}{dP} > 0 \text{ for } |\lambda| > 1$$

and, for $|\lambda| < 1$,

$$\frac{dW}{dP} < 0 \text{ only if } X^2_L >> X^2_U$$

From (16) it is clear that for wage inequality to increase ($\frac{dW}{dP} < 0$), marginal productivity of skilled labour must be higher than for unskilled labour. In general, we would expect this to hold in most developing countries. If this difference is sufficiently high then wage inequality could increase.

Proposition 2
In small open economies with a large body of unskilled labour force specific to the export sector, trade liberalization can lead to a decline in the relative wages or increasing wage inequality only if the skilled labour is more productive than unskilled labour.

This brings out the general point that the issue is not one of trade liberalization per se but the differing productivities of skilled and unskilled labour. It is the latter that leads to increasing wage inequality rather than trade liberalization per se. For countries like India exports tend to be dominated by the unorganized sector with a preponderance of unskilled labour. The hypotheses of Propositions 1 and 2 seem to be confirmed by some recent empirical evidence. Thus, Hashim and Banga (2009) in a study for India show that in the period 1998-2005, increased trade has been accompanied by increasing wage inequality despite increase in the the wages of both skilled and unskilled labour. Our results show that this is probably due to the huge gap in productivity between skilled and unskilled labour in the export sector.

Thus, we are able to construe a relative wage hypothesis within an absolute wage hypothesis where one is not necessarily in conflict with the other.

The coexistence of rising wage inequality in the presence of increasing absolute wages of both skilled and unskilled labour allows us to derive our third important proposition:

**Proposition 3**

Trade liberalization may lead to rising wage inequality. However, this does not imply that the absolute level of poverty would also increase and it may in fact decline.

Hence, it may still be true that there is increasing discontentment with trade due to falling relative wages. This is a bit like a Dussenberry effect: even though absolute real wages of all labour is going up, the increasing wage inequality is leading to some discontent. This probably explains the current dissatisfaction with trade liberalization in many developing countries.

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2 This sector comprises products like textiles, handicrafts etc. In recent years exports of the Information Technology (IT) related services have been increasing at a rapid pace and constituted 50 percent of service exports. However, even here the dominant category are Business Process Outsourcing (BPO) services where unskilled labour dominates. (see, Banga and Kumar (op.cit).)
Section 2:

Model 2

As in Model 1, we assume the country is a small one which takes world prices of tradable as given. There are two sectors producing final goods $X_1$ and $X_2$. Once again we call them Manufacturing and Services. However, now there is a third sector, for example agriculture, producing an intermediate good, $M$ which is used in the production of both $X_1$ and $X_2$. $M$ is also a non-traded good\(^3\). There are two factors of production $L_S$ and $L_u$. However, while $L_S$ is used in the production of all three sectors, $L_u$ (unskilled labour) is specific to the sector, $M$. The total supply of unskilled labour, $\bar{L}_u$ is given.

The model then is,

\[ X_1 = X_1 (L_1^s, M_1) \]  
\[ X_2 = X_2 (L_2^s, M_2) \]  
\[ M = M (L_m^s, L_u) \]

Hence

\[ C_{m1} X_1 + C_{m2} X_2 = M \]  
\[ C_{Lu} M = L_u \]

Where the $C_{ij}$’s, the ith factor required per unit of the jth sector, or, in other words are the input output coefficients.

\(^3\) For some countries like India there is evidence to show that, due to domestic consumption requirements, most basic agricultural commodities are largely non-traded. Exports of most are subject to clearance by the authorities and in large production items like wheat, milk etc. periodically export restrictions are clamped for domestic political reasons.
The full employment conditions are,

\[ C_{L1} X_1 + C_{L2} X_2 + C_{LM} M = \bar{L}_s \quad (22) \]

and

\[ L_u = \bar{L}_u \quad (23) \]

Perfect mobility of skilled labour implies that the skilled wage, \( W_s \), is equalized across sectors so that

\[
\begin{align*}
W_s &= P_1 MP_1^{Ls} = P_2 MP_2^{Ls} \\
&= P_m MP_m^{Ls}
\end{align*}
\]

(24) – (26)

The determination of the wage of the specific factor, \( w_u \), is given by

\[ W_u = P_m MP_m^{\mu_u} \quad (27) \]

As is usual, we need a numeraire good. So

\[ P_1 = 1 \quad (28) \]

Equations (17)-(28) comprise the 12 equations of our model and the 12 endogenous variables are the \( X \)'s (2), \( L_s \)'s (3), \( L_u \), \( M \), \( M_1 \), \( M_2 \), \( W_s, W_u \) and \( P_M \). Hence, some solution exists. Parameters are \( \bar{L}_s \), \( \bar{L}_u \) and price \( P_2 \) of commodity 2.

Using equations (20), (21) and substituting from other equations in the model we can reduce the model to a system of 3 equations in \( X_1, X_2 \) and \( M \).
Here equations (29) – (31) can be solved for the three variables of the model, $X_1$, $X_2$ and $M$. By appropriate substitution in the model the other variables can be solved for.

### 2.1. Comparative Statics Results

Total differentiation of (29) – (31) w. r. t $L_S$, under the assumption of constant commodity and factor prices, gives us

\[
\begin{align*}
\frac{dX_1}{dL_S} & \geq 0 \iff \frac{L_2}{M_2} \leq \frac{L_1}{M_1} \\
\frac{dX_2}{dL_S} & = 0 \iff \frac{L_2}{M_2} = \frac{L_1}{M_1} \\
\frac{dM}{dL_S} & = 0
\end{align*}
\]

From (32) it is clear that an increase in the supply of skilled labour (with unchanged commodity prices) leads to an increase in the output of commodity ($X_2$) which uses skilled labour intensively relative to the intermediate good and reduces the output of the other commodity, $X_2$. Hence, the Rybczynski theorem holds. However, the output of the intermediate good remains unchanged as shown in the third equation in (32).

### 2.1. Wage Inequality

In this paper the focus is on what happens to wage inequality as this economy increases trade. We assume again that commodity 2 is the exported commodity so that the consequence of

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4 See Appendix B for detailed derivations
trade is a parametric increase in \( P_2 \). This parametric variation is applied to the model developed above.

The price equations of the model can be written as

\[
C_{L1} W_S + C_{M1} P_M = 1 \tag{33}
\]

\[
C_{L2} W_S + C_{M2} P_M = P_2 \tag{34}
\]

And

\[
C_{LM} W_S + C_{LU} W_U = P_M \tag{35}
\]

Where \( P_1 \) equals 1 from equation (28).

Substituting for \( P_M \) from (35) into (33) and (34) and totally differentiating (33) and (34) assuming that the price of the second good only changes, we get,

\[
\theta_1 W^*_S + \theta_2 W^*_U = 0 \tag{36}
\]

\[
\theta_3 W^*_S + \theta_4 W^*_U = P^*_2 \tag{37}
\]

And the \( \theta \)'s are the four elements of the coefficient matrix, \( \theta \), of the system of equations (36) and (37) which solve for \( W^*_S \) and \( W^*_U \), the relative changes in skilled and unskilled wages respectively. In the appendix we show that

\[
|\theta| \geq 0 \iff \frac{L_2}{M_2} < \Rightarrow \frac{L_1}{M_1} \tag{38}
\]

that is, its sign depends on the relative intermediate good to skilled labour intensity of the two commodities, \( X_1 \) and \( X_2 \). Thus, from (36) and (37) we get

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5 Since the intermediate good itself depends on \( L_S \) and \( L_U \), this condition is equivalent to comparing the direct and indirect requirements of \( L_S \) and \( L_U \) in the production of \( X_1 \) and \( X_2 \).
\[
W^*_{S}/P^*_2 = -[\theta_2]/\theta_1 \quad \text{and} \quad W^*_{U}/P^*_2 = [\theta_1]/\theta_1 \quad \text{so that}
\]

\[
\frac{dW_S}{dP_2} \geq 0 \iff \frac{L_2}{M_2} \geq \frac{L_1}{M_1}
\]

and

\[
\frac{dW_U}{dP_2} \geq 0 \iff \frac{L_2}{M_2} \leq \frac{L_1}{M_1}
\]  \hspace{1cm} (39)

From (39) we see that if the export sector, sector 2, is relatively intensive in the use of the intermediate good relative to skilled labour (that is \(L_1/M_1 > L_2/M_2\) so that \(\theta > 0\)) then wages of skilled workers falls and that of unskilled labour increases when this country’s trade increases. In general, the relative wage, \(W\), unambiguously falls or rises depending on the relative intensity of use of skilled labour and the intermediate good in the production of the two commodities.

**Proposition 4.**

*In small countries with a large unskilled labour force specific to the non-traded intermediate goods sector, increased trade leads to an increase (decrease) in wage inequality if the export sector is more (less) intensive in the use of the intermediate good. The decrease (increase) of inequality is accompanied by an increase (decrease) in wages paid to unskilled labour and a decrease (increase) in wages paid to skilled labour.*

This, of course, is the well known Stolper-Samuelson result but with a twist. An increase in the price of \(X_2\) increases the return to the factor \(M\) which is relatively intensively used in its production. The return to \(M\) is given by \(P_M\) since \(M\) is also a produced commodity. Thus \(P_M\) increases. Then from (37), since \(W_S^*\) is negative, it must be true that \(W_U^*\) is positive. Therefore, wage inequality declines. However, in contrast to the result in Batra and Beladi (op.cit),our results on wage inequality are not conditional and only require that products must have differing input intensities.
In a study for India, Banga and Kumar (op.cit.) have shown that the fastest growing exports are of the service sector where 50 percent is made up of IT services, specifically, software services. However, at $ 40 billion in 2008-09, these constitute less than 1 percent of the GDP of about $ 4.5 trillion. Hence, it is unlikely that even expanded exports of the BPO sector (where unskilled labour dominates) would make any dent on wage inequality. The problem seems to be stagnant exports of the small manufacturing sector where most of the unskilled agricultural labour could be absorbed.

Our model thus provides lessons for other developing countries like India. It is not that trade increases wage inequality and poverty but the fact that the export sector does not extensively employ products of the intermediate good (for example, agricultural goods) sector where most of the poor and unskilled labor is employed. Our model thus makes a case for developing a manufacturing sector based on exports of processed agricultural goods. In the current scenario where the growth of exports in India are based largely on a service sector which has few linkages to agriculture, trade is likely to lead to an increase in wage inequality and absolute poverty.

Section 4:

Conclusion

Empirical studies by and large reject the most important hypothesis of the Heckscher-Ohlin-Samuelson (HOS) model that trade will improve both inter and intra country wage inequality. Theoretical studies using general equilibrium models have tried to explain this by arguing that the issue of inequality is not linked to trade. Other studies argue that the basic assumptions of the HOS model are violated so that its predictions are not observed. A third set of such studies have modified the HOS model to include specific factors and intermediate goods but are unable to get unconditional results on wage inequality. That wage inequality could increase with trade thus seems to be best explained by using imperfect competition models of trade and product heterogeneity. Finally, there is the theoretical issue of reconciling the absolute wage
hypothesis with the relative wage hypothesis within the standard HOS framework. The last step is necessary to reconcile observed empirical trends with theoretical models.

In this paper we have attempted to set up two models and specifically model wage inequality. The simple model with specific factors indicates that the basis for inequality is differing productivities of skilled and unskilled labour. The model also suggests that while trade may increase wage inequality this does not imply that poverty increases as wage of unskilled workers also increase.

We also set up a modified HOS model which includes specific factors and a non-traded good which is itself an input into the production process. The model gives unambiguous results on the impact of trade on wage inequality which depend on the link between the non-traded good and the export sector. The weaker this link the more likely that trade will result in increased wage inequality. This suggests that what is relevant is the structure of trade and not trade per se.

In many developing countries today there is considerable concern over the growing wage inequality as global trade expands. We suggest that the solution here to be found in domestic policies like productivity improvements and linkages of non-trade and trade goods sectors. Protective trade policies are not a solution.
Appendix A

Model 1

From Equations (6) and (7) in the text and using (3) – (5), we get

\[ X^1_{L_1} (\bar{K}, L_1) = p X^2_{L_2} (L_2, \bar{L}_u) \]  \hspace{1cm} (A.1)

Totally differentiating (A.1) above and given that \( \bar{K} \) and \( \bar{L} \) are fixed, we get,

\[ \left( \frac{\delta X^1_{L_1}}{\delta L_1} \right) dL_1 = \left( p \frac{\delta X^2_{L_2}}{\delta L_2} \right) dL_2 + X^2_{L_2} (L_2, \bar{L}_u) d\bar{p} \]

Or

\[ \left( \frac{\delta X^1_{L_1}}{\delta L_1} \right) dL_1 - \left( p \frac{\delta X^2_{L_2}}{\delta L_2} \right) dL_2 = X^2_{L_2} (L_2, \bar{L}_u) d\bar{p} \]  \hspace{1cm} (A.2)

From equations (3) –(5) in the text

\[ dL_1 + dL_2 = 0 \]

Or,

\[ dL_1 = -dL_2 \]  \hspace{1cm} (A.3)

Now, using (A.3) in (A.2), we get
\[- \left[ \frac{\delta X^1_L}{\delta L_1} + p \frac{\delta X^2_L}{\delta L_2} \right] dL_2 = X^2_L(L_2, \bar{L}_a) d\bar{p} \]

Or

\[ \frac{dL_2}{d\bar{p}} = - \frac{X^2_L(L_2, \bar{L}_a)}{\left[ \delta X^1_L / \delta L_1 + p \delta X^2_L / \delta L_2 \right]} \]

Or,

\[ \frac{dL_2}{d\bar{p}} = - \frac{X^2_L(L_2, \bar{L}_a)}{[X^1_{1L} + p X^2_{2L}]} > 0 \quad (A.4) \]

(Since, by the concavity property of the production function both \(X^1_{1L}\) and \(p X^2_{2L}\) are negative).

Therefore,

\[ \frac{dL_1}{d\bar{p}} < 0 \]

So, from (A.3)

Again, totally differentiating the equations (6) – (7) in the text we get,

\[ dW^S = X^1_{1L} dL_1 = p X^2_{2L} dL_2 + X^2_L dP \]

Therefore,

\[ dW^S / d\bar{p} = X^1_{1L} dL_1 / d\bar{p} = p X^2_{2L} dL_2 / d\bar{p} + X^2_L \quad (A.5) \]

So, from (A.4), and the fact that \(X^1_{1L} < 0\),

\[ X^1_{1L} dL_1 / d\bar{p} > 0 \text{ implies } dW^S / d\bar{p} > 0 \]

The equality relation of (A.5) requires that the expression
\[ p X_{LL}^2 \frac{d L_2}{dp} + X_L^2 > 0 \]

must be positive.

Similarly, totally differentiating Equation (8) in the text we get

\[ d W/U dp = X_U^2 + p X_{UL}^2 \frac{d L_2}{dp} > 0 \quad (A.6) \]

( Since, \( X_U^2 > 0 \) being the marginal productivity of unskilled labour and \( d L_2/ dp > 0 \) by (A.4) and \( X_{UL}^2 > 0 \) since more of skilled labour on a given specific unskilled labour raises the productivity of unskilled labour)

Now, we define, \( W = W^U/W^S \),

\[ \frac{d W}{dp} = \frac{d W^U}{dp} - \frac{d W^S}{dp} = X_U^2 + p X_{UL}^2 \frac{d L_2}{dp} - \left\{ p X_{LL}^2 \frac{d L_2}{dp} + X_L^2 \right\} \quad (A.7) \]

Substituting from (A.4) for \( d L_2/ dp \), we write the above expression as,

\[ = X_U^2 - X_L^2 + \left\{ p X_{UL}^2 - p X_{LL}^2 \right\} \frac{-X_L^2}{(X_{LL}^1 + p X_{LL}^2)} \]

\[ = X_U^2 - X_L^2 (1+\lambda) \quad (A.8) \]

Where \( \lambda = \left\{ p X_{UL}^2 - p X_{LL}^2 \right\}/\left( X_{LL}^1 + p X_{LL}^2 \right) < 0 \) (since marginal products are diminishing)

These results are shown in Equation (15) in the text.
Rybczynski effect:

When \( L_S \) increases

At constant commodity price, we can write from (A.2)

\[
(\delta X^1_L / \delta L_1) dL_1 = p (\delta X^2_L / \delta L_2) dL_2
\]  
(A.9)

From full employment equation (3) in the main text

\[
dL_1 + dL_2 = dL
\]

Dividing both sides of (A.9) by \( dL \) we get

\[
\frac{\delta X^1_L}{\delta L_1} \frac{dL_1}{dL} = p \frac{\delta X^2_L}{\delta L_2} \frac{dL_2}{dL} 
\]  
(A.10)

Since, marginal productivities are positive it follows from (A.10) that

\[
dL_1 / dL > 0 \text{ and } dL_2 / dL > 0
\]
Appendix B:

Model 2

Comparative Statics:

\textit{Rybczynski Effect:}

Equations (29)-(31) in the text can be written as

\[
\begin{pmatrix}
C_{L1} & C_{L2} & C_{LM} \\
0 & 0 & C_{Lu} \\
C_{m1} & C_{m2} & -1
\end{pmatrix}
\begin{pmatrix}
X_1 \\
X_2 \\
M
\end{pmatrix}
=
\begin{pmatrix}
E_s \\
\bar{L}_U \\
0
\end{pmatrix}
\]  
(B.1)

Or

\[
C \ V = Z
\]  
(B.2)
Where $C$ is the coefficient matrix, $V$ is a column matrix representing the variables in the system and $Z$ is column matrix representing the parameter of our system. Thus,

On simplification, we get,

\[ ICI = C_{M1} \cdot C_{M2} \cdot C_{Lu} \left( L_2/M_2 - L_1/M_1 \right) \]

or

\[ C \gtrless 0 \text{ as } L_2/M_2 \gtrless L_1/M_1 \] (B.3)

In other words, if $X_2$ is relatively skilled labour (intermediate good) intensive compared to $X_1$ then $C$ determinant is positive(negative). Here, in deriving (B.3) the commodity prices are assumed to be unchanged.

Solution of (B.1) and (B.2) and differentiation with respect to changes in factor supplies under the assumption of unchanged prices gives us

\[ \frac{dX_1}{d\Gamma_a} = \frac{1}{ICI} \left[ -C_{Lu} \cdot C_{M1} \right] \gtrless 0 \quad \text{iff} \quad C \gtrless 0 \]

Similarly we have

\[ \frac{dX_2}{d\Gamma_a} = \frac{1}{ICI} \left[ C_{Lu} \cdot C_{M1} \right] \gtrless 0 \quad \text{iff} \quad C \gtrless 0 \]

And

\[ \frac{dM}{d\Gamma_a} = 0 \]

These results are shown in Equation (32) in the main text.

**Stolper – Samuelson effect:**

Equations (36)-(37) in the text can be written under the assumption of changes in factor prices and change in the second commodity prices in the matrix form as
It may be noted that the sign of the determinant again depends on the sign of the C determinant. However, C and \( \theta \) have opposite sign. It is clear that the \( \theta \) determinant gives the direct and indirect requirements of skilled and unskilled labour. Assuming \( P_1^* = 0 \), we can determine the effect of a rise in \( P_2^* \) on the factor rewards. Thus, the effect on the skilled wage rate is

\[
\begin{align*}
W_S^* &= \begin{pmatrix}
0 & \Theta_{M1} \Theta_{LU} \\
P_2^* & \Theta_{M2} \Theta_{LU}
\end{pmatrix} / \Theta \\
W_S^*/P_2^* &= -\frac{\Theta_{M1} \Theta_{LU}}{\Theta} \quad \text{B - 6}
\end{align*}
\]
\[ W_S'/P_2' \geq 0 \text{ as } C \geq 0 \text{ and } \Theta \leq 0 \]

In the same way, we can find the effect of a rise in \( P_2 \) on the unskilled wage rate:

\[ W_U'/P_2' = (\Theta L_1 + \Theta M_1)/\Theta \]

Therefore,

\[ W_U'/P_2' \leq 0 \text{ as } C \leq 0 \text{ and } \Theta \leq 0 \]

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