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Intelligent minority game with genetic crossover strategies

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Abstract. We have developed a novel game theoretical model of N interacting agents playing a minority game such that they change their strategies intelligently or adaptively depending on their temporal performances. The strategy changes are done by generating new strategies through one-point genetic crossover mechanism. The performances of agents are found to change dramatically (from losing to winning or otherwise) and the game moves rapidly to an efficient state, in which fluctuations in the number of agents performing a particular action, characterized by the variance σ^2, reaches a low value.

PACS. 87.23.Ge Dynamics of social systems – 02.50.Le Decision theory and game theory – 87.23.Kg Dynamics of evolution

1 Introduction

The dynamics of interacting agents competing for scarce resources are believed to underlie the behaviour of various complex systems of natural, social and economical origin [1–5]. An example of such a complex system is financial market, where competing agents interact with each other and try to perform their best in order to survive in accordance with the idea of "survival of the fittest" in biology. However, the agents need not be human beings restricted to the market place but could vary in form, size and nature. Also the behavioral patterns of the agents could vary– the agents are said to be "heterogeneous". In many studies of such market behaviour, tools of statistical physics have been combined with theories of economics [6–9], like game theory, which deals with decision making of a number of rational opponents under conditions of conflict and competition [10–15]. However, conventional economics studies consistent patterns in behavioral equilibrium that require no further interaction. Since complex systems are constantly evolving processes the patterns which they create are, in general, out-of-equilibrium and hence beyond the scope of conventional economical analyses. Also, in traditional economics the "rational expectations" approach is assumed to be valid, which in reality may not hold altogether.

In this paper, we develop a game theoretical model of a large number of heterogeneous interacting agents adapting periodically to changing situations such that we can have a better understanding of the behaviour of the complex systems such as financial markets. Our model is based on the minority game [11], which provides an alternative to the common approach of microeconomics with a single representative agent, based on the assumption that all the agents are identical [16]. The minority game model consists of agents having a finite number of strategies and finite amount of public information, interacting through a global quantity (whose value is fixed by all the agents) representing a market mechanism. In the original model the agents choose their strategies through a simple adaptive dynamics based on *inductive reasoning* [5]. Here, we introduce the fact that the agents are *intelligent* or *adaptive* and in order to be best or survive in the market, modify their strategies periodically depending on their performances. For modifying the strategies, we choose the mechanism of *one-point genetic crossover*, following the ideas of genetic algorithms in computer science and operations research. In fact, these algorithms were inspired by the processes observed in natural evolution [17–19] and it turned out that they solve some complicated problems without knowledge of the decoded world. In nature, one-point crossover occurs when two parents exchange parts of their corresponding chromosomes after a selected point, creating offsprings [19]. We then study this simple model of complex adaptive systems and examine the behavioral patterns of the agents.

2 Model and results

The basic minority game consists of an odd number of agents N who can perform only two actions, at a given time t, and an agent wins the game if it is one of the members of the minority group at the end of the game and then time t increases by unity. The two actions, *i.e.* "buying" or "selling" commodities, are denoted here by 0 or 1, respectively. Further, it is assumed that all the agents

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have access to finite amount of public information, which is a common bit-string "memory" of the M most recent outcomes. Thus the agents are said to exhibit "bounded rationality" [5]. For example, in case of memory $M = 2$ there are $P = 2^M = 4$ possible "history" bit strings: 00, 01, 10 and 11. A "strategy" consists of a response, *i.e.*, 0 or 1, to each possible history bit strings; therefore, there are $G = 2^P = 16$ possible strategies which constitute the "total strategy space". At the beginning of the game, each agent randomly picks k strategies, and after a game, assigns one "virtual" point to the strategies which would have predicted the correct outcome; the best strategy is the one which has the highest virtual point. The performance of the player is measured by the number of times the player wins, and the strategy, which the player uses to win, gets a "real" point. We also keep a record of the number of agents who have chosen at time t a particular action, say, "selling" denoted by $N_1(t)$. The fluctuations in the behaviour of $N_1(t)$ indicate the total utility of the system. For example, we may have a situation where only one player is in the minority and thus wins, and all the other players lose. The other extreme case is when $(N-1)/2$ players are in the minority and $(N+1)/2$ players lose. The total utility of the system is highest for the latter case as the total number of the agents who win is maximum. Therefore, the system is more efficient when the fluctuations around the mean are smaller than when they are larger. These fluctuations can be characterized by the variance σ^2 such that smaller values of σ^2 would correspond to the system being in a more efficient state.

In our model, the players of the basic minority game are assumed to be intelligent by modifying their strategies at regular time-intervals τ depending on their current performances. If they find that they are among the fraction n (where $0 < n < 1$) of the worst performing players, they modify any two of their strategies chosen randomly from the pool of k strategies and use one of the new strategies generated. The mechanism by which they modify their strategies is that of one-point genetic crossover illustrated schematically in Figure 1. Here the strategies s_i and s_j act as the parents and by choosing the breaking point in them randomly, and performing one-point genetic crossover, the children s_k and s_l are produced and substitute the parents.

It should be noted that the strategies are changed by the agents themselves and even though the strategy space evolves, it is still of the same size and dimension; thus considerably different from earlier models [11,20,21]. Challet *et al.* [11] generalized the basic minority game mentioned above to include the Darwinist selection: the worst player is replaced by a new one after some time steps, the new player is a clone of the best player, *i.e.* it inherits all the strategies but with corresponding virtual capitals reset to zero. To keep a certain diversity they introduced a mutation possibility in cloning. They allowed one of the strategies of the best player to be replaced by a new one. Since strategies are not just recycled among the players any more, the whole strategy phase space is available for selection. They expected this population to be capable of "learning" since bad players were weeded out with time,

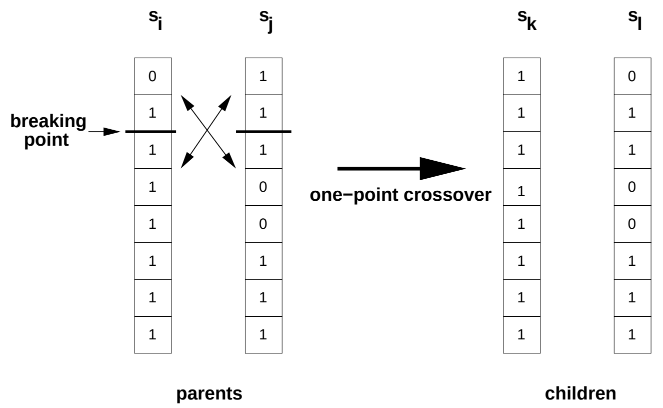

Fig. 1. Schematic diagram to illustrate the mechanism of one-point genetic crossover for producing new strategies. The strategies s_i and s_j are the parents. We choose the breaking point randomly and through this one-point genetic crossover, the children s_k and s_l are produced and substitute the parents.

and fighting was among the so-called "best" players. Indeed, they observed that the learning emerged in time though very slowly. Fluctuations were reduced and saturated, which implied that the average gain for everybody did improve but never reached the "ideal limit". Li *et al.* [20] studied the minority game in the presence of evolution, where games were played with different values of m and different numbers of agents, analogous to that found in the non-evolutionary, adaptive games. Li *et al.* [21] also studied evolution in minority games by examining games in which agents with poorly performing strategies can trade in their strategies for new ones from a different strategy space. There have been several other variants of the minority game [22] but the mechanisms of evolution proposed earlier and objectives of studies are clearly different from the mechanism we present here.

In our study, we pick up from the total strategy space only *uncorrelated* strategies (in our case, strategies which have pairwise Hamming distance $d_H = 0.5$ and the average over all the strategies is also 0.5) [23]. Though this choice is not really necessary, our intention is to keep the average Hamming distance in an agent's pool and thus the average Hamming distance in the whole system equal to 0.5 throughout the evolution of the game. Our definition of average Hamming distance should not be confused with other commonly used definitions, because we have first taken the average over the distances in an agent's strategy pool, and then taken the average for all the agents. The crossover mechanism which we have used here does not alter the average Hamming distance. Other ways to generate new strategies (see *e.g.*, [24,25]) alter the average Hamming distance and the study of the time evolution of the average Hamming distance is found to be very interesting.

In Figure 2, the performances of the players of our model are compared with those of the basic minority game. We have scaled the performances of all the players such that the mean is zero for easy comparison of the relative success of the agents in each case. In this figure

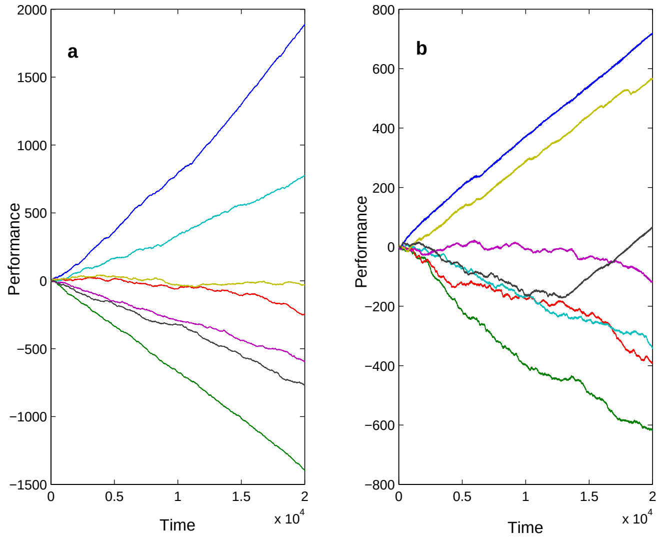

Fig. 2. Plots of the performances of the best player, the worst player and five randomly selected players in (a) the basic minority game with $N = 1001$, $M = 5$, $k = 10$ and $t = 20\,000$, and (b) in our intelligent minority game with $N = 1001$, $M = 5$, $k = 10$, $t = 20\,000$, $n = 0.3$ and $\tau = 100$.

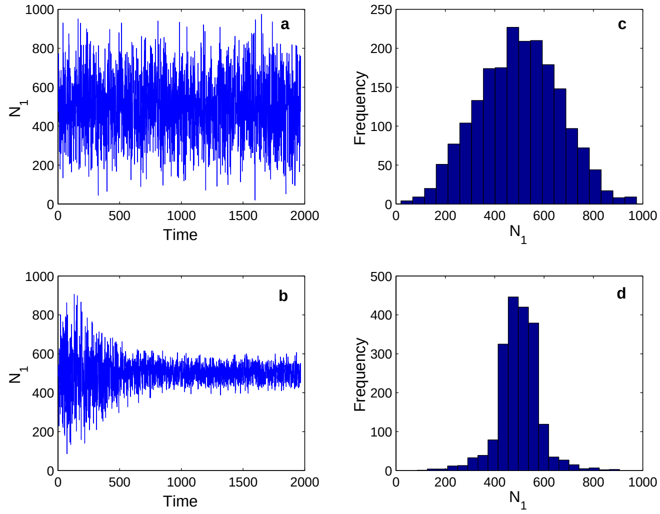

Fig. 3. Plots of the (a) time-variation of N_1 for the basic minority game, (b) time-variation of N_1 for the intelligent minority game, and (c) histogram of N_1 for the basic minority game and (d) histogram of N_1 for the intelligent minority game. Simulations of the basic minority game were performed with $N = 1001$, $M = 5$, $k = 10$ and $t = 1999$ and of the intelligent minority game with $N = 1001$, $M = 5$, $k = 10$, $t = 1999$, $n = 0.3$ and $\tau = 100$.

it is clearly evident that there are significant differences in the performances of the players. The performance of a player in the basic minority game does not change drastically in the course of the game as seen in Figure 2a. However, like in most evolutionary models, in our model too the performances of the players may change dramatically even after initial downfalls, and agents often do better after they have produced new strategies with the one-point genetic crossovers, as illustrated in Figure 2b.

In order to study the efficiency of the game, we plot the time-variation of N_1 for the basic minority game in comparison to our model in Figures 3a and b, respectively. Also the corresponding histograms of N_1 for the basic minority game and our model are plotted in Figures 3c and d. Clearly evident from these figures is the fact that when we allow one-point genetic crossovers in strategies, the system moves toward a more efficient state. This is because the fluctuations in N_1, which is seen in the histogram of N_1 becoming narrower and sharper. We have also studied the effect of increasing the fraction of players n on the distributions of the number of switches and the number of genetic crossovers the players make. The results in Figure 4 illustrate the fact that as n increases, more players have to make large number of strategy switches and crossovers in order to improve their performances.

We have calculated the variance σ^2 of N_1 and plotted σ^2/N *versus* the parameter $2^M/N$ in Figure 5a, σ^2/N *versus* M in Figure 5b. We have found that the quantity σ^2/N, as the mean of characterizing the behaviour of the game, may not always be useful since in the framework of genetic crossovers, it is possible to reach the "ideal limit" where the fluctuations totally disappear and hence $\sigma^2/N = 0$ [24,25]. Following the convention to plot the σ^2/N *versus* the parameter $2^M/N$, in Figure 5a we show

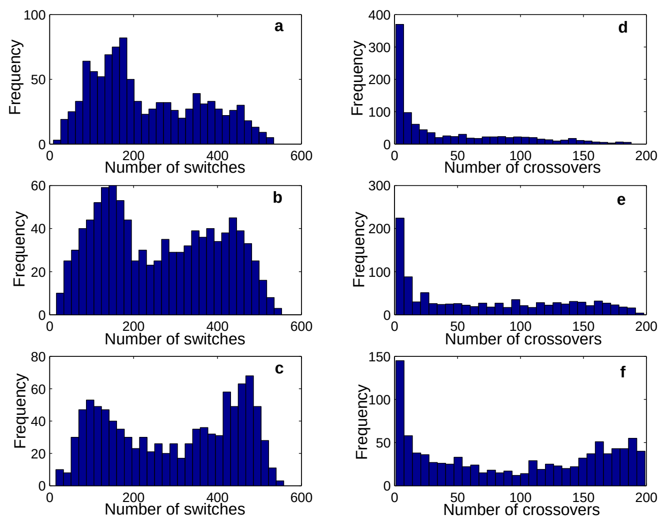

Fig. 4. Histograms of the number of switches the players make in the intelligent minority game for (a) $n = 0.3$ (b) $n = 0.4$ (c) $n = 0.5$, and histograms of the number of genetic crossovers the players make in the intelligent minority game for (d) $n = 0.3$ (e) $n = 0.4$ and (f) $n = 0.5$. The simulations have been made with $N = 1001$, $M = 4$, $k = 10$, $t = 1999$ and $\tau = 10$.

the behaviour of σ^2/N *vs.* $2^M/N$ for both the original minority game and our intelligent minority game model by varying M and N, when the pool of strategies is set to $k = 2$. Here it is seen that for the basic minority game there is a minimum of σ^2/N at $2^M/N \approx 0.5$, but for the present set of parameters there is no such minimum for the intelligent minority game.

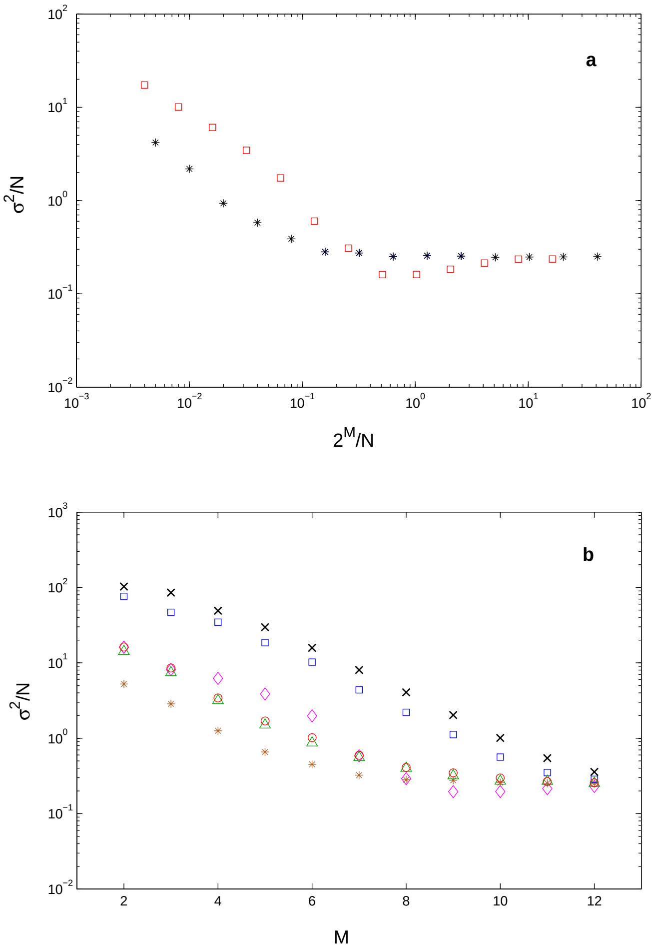

Fig. 5. (a) The plot of σ^2/N against the parameter $2^M/N$ for $k = 2$, by varying M from 2 to 11 and N from 25 to 1001 for the basic minority game (squares) and the intelligent minority game (asterisk marks). Simulations were made for $t = 5000$ and averaging over ten different samples in each case. The parameter values chosen for the intelligent minority game were $\tau = 10$ and $n = 0.5$. (b) The plot of σ^2/N against M for different values of k for the basic minority game and the intelligent minority game. For the basic minority game, we have studied the cases of $k = 2$ (diamonds), $k = 6$ (squares) and $k = 10$ (cross marks). For the intelligent minority game, we have studied the cases of $k = 2$ (asterisk marks), $k = 6$ (triangles) and $k = 10$ (circles). The simulations for the basic minority game have been made with $N = 1001$ and $t = 5000$, and for the intelligent minority game have been made with $N = 1001$, $t = 5000$, $n = 0.5$ and $\tau = 10$, and averaged over five different samples in each case.

It should be noted that when $k = 2$, the crossover mechanism is not very effective because an agent cannot evolve good strategies being forced to make the crossover and generate totally new strategies after every time interval τ. Thus the basic minority game is better for $k = 2$ around the minimum. We intuitively expect that the mechanism is more effective for other combinations of the parameters k, n and τ.

We also plot in Figure 5b the quantity σ^2/N as a function of M ranging from 2 to 12 for both games when $N = 1001$ and for different values of k. It is seen that when the number of strategies, *i.e.* k, is increased the efficiency of the original minority game rapidly decreases while at the same time making the curves monotonically decreasing. However, in the case of the intelligent minority game, the situation of curves monotonically decreasing remains for any combinations of k, M and N, we have studied with the current parameter set. Also we found that as the value of k is increased, the efficiency decreases, but at a rate much less than in the basic minority game. For both games, the values of σ^2/N seem to converge towards a common value for large values of M. If we compare the two games, we find that for large k values and moderate values of M, the differences in σ^2/N is very large, thus rendering the intelligent minority game market much more efficient.

Furthermore, we have observed that in our model, the worst players were often those who switched strategies most frequently while the best players were those who made the least number of switches after finding a good strategy (for related work in switching strategies *cf.* Ref. [11]). In addition, we have found that the players who do not make any genetic crossovers are unable to compete with those who make genetic crossovers, and their performances were found to fluctuate around the zero mean. It was also found that as the time-interval τ between consecutive crossovers is increased, the time for the system to reach an efficient state increases. The detailed studies and analysis of results for various combinations of parameters have been studied in another communication [24].

3 Discussions

One advantage of our model is clearly that the dimensionality of the strategy space as well as the number of elements in the strategy space remain the same. It is also appealing that starting from a small number of strategies, many "good" strategies can be generated by the players in the course of the game. Even though the players may not have performed well initially, they often did better when they used new strategies generated by the one-point genetic crossovers. Finally, it should be pointed out that even in the framework of genetic algorithms, there are various ways to generate new strategies. One possibility is that we make a one-point genetic crossover between the two worst strategies and replace the parents by the children. Another possibility is to make "hybridized genetic crossover", where the one-point genetic crossover is made between the two best strategies, replace the worst two strategies with the children and retain the parents as well. We have studied some of these modifications in another communication [25]. This general method described in the paper is very simple and powerful and it can lead to further studies not only in game theory in economics but also form the basis of widely varying topics such as the modelling of biological evolution of species [26].

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References

1. G. Parisi, Physica A **263**, 557 (1999)
2. B.A. Huberman, P.L.T. Pirolli, J.E. Pitkow, R.M. Lukose, Science **280**, 95 (1998)
3. M. Nowak, R. May, Nature **359**, 826 (1992)
4. T. Lux, M. Marchesi, Nature **397**, 498 (1999)
5. W.B. Arthur, Am. Econ. Rev. **84**, 406 (1994)
6. R.N. Mantegna, H.E. Stanley, *An Introduction to Econophysics* (Cambridge University Press, Cambridge, 2000)
7. J.-P. Bouchaud, M. Potters, *Theory of Financial Risk* (Cambridge University Press, Cambridge, 2000)
8. S. Moss de Oliveira, P.M.C. de Oliveira, D. Stauffer, *Evolution, Money, War and Computers* (B.G. Teubner, Stuttgart-Leipzig, 1999)
9. J.D. Farmer, IEEE Comp. Sci. Eng. Nov.-Dec., 26 (1999)
10. R. Myerson, *Game Theory: Analysis of Conflict* (Harvard University Press, Cambridge, Massachusetts, 1991)
11. D. Challet, Y.-C. Zhang, Physica A **246**, 407 (1997)
12. D. Challet, M. Marsili, R. Zecchina, Phys. Rev. Lett. **84**, 1824 (2000)
13. R. Savit, R. Manuca, R. Riolo, Phys. Rev. Lett. **82**, 2203 (1999)
14. A. Cavagna, J.P. Garrahan, I. Giardina, D. Sherrington, Phys. Rev. Lett. **83**, 4429 (1999)
15. D. Lamper, S.D. Howison, N.F. Johnson, Phys. Rev. Lett. **88**, 17902 (2002)
16. A. Mas-Colell, M.D. Whinston, J.R. Green, *Microeconomic Theory* (Oxford University Press, New York, 1995)
17. J.H. Holland, *Adaptation in Natural and Artificial Systems*, (University of Michigan Press, Ann Arbor, 1975)
18. D.E. Goldberg, *Genetic Algorithms in Search, Optimization and Machine Learning* (Addison-Wesley, Reading, Massachusetts, 1989)
19. *Handbook of Genetic Algorithms*, edited by D. Lawrence (Van Nostrand Reinhold, New York, 1991)
20. Y. Li, R. Riolo, R. Savit, Physica A **276**, 234 (2000)
21. Y. Li, R. Riolo, R. Savit, Physica A **276**, 265 (2000)
22. `http://www.unifr.ch/econophysics/minority`
23. D. Challet, Y.-C. Zhang, Physica A **256**, 514 (1998)
24. M. Sysi-Aho, A. Chakraborti, K. Kaski, submmitted to Phys. Rev. E, preprint available at `cond-mat/0305283` (2003)
25. M. Sysi-Aho, A. Chakraborti, K. Kaski, Physica A **322**, 701 (2002)
26. M. Sysi-Aho, A. Chakraborti, K. Kaski, J. Kertész, in preparation (2002)