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Sujata Basu

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Jawaharlal Nehru University

India

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Sujata Basu $^{\rm 1}$

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¹Ph.D Scholar, Center for International Trade and Development (CITD), School of International Studies (SIS), Jawaharlal Nehru University (JNU), New Delhi, INDIA

Abstract

An endogenous skilled biased growth model has been considered to show that along the growth path wage gap widened and both upward and downward mobility fall. This implies that education becomes more correlated with initial conditions and less related with the cognitive ability. Growth occurs through the twin channels of technology – imitating from the world technology frontier and innovating on its own technology level – innovation being more skilled-intensive than imitation. An imperfect capital market has been considered where individual's education decision depends on the cognitive ability as well as on the parental income. Moreover, it is shown that growth enhancing education policy leads to absolute convergence of all the economies to the world technology frontier. In the imitation-innovation regime, life time utility gap within skilled as well as unskilled human capital rise due to parental income differences. Furthermore, life time utility gap within skilled human capital rises due to cognitive ability differences.

Journal of Economic Literature Classifications: E02, I24, I25, JHO, O11, O31, O32, O40. Key Words: Economic Growth, Endogenous Labor Composition, Intergenerational Mobility, Inequality, Imitation-Innovation, Convergence.

Address correspondence to: Sujata Basu, Room No 219, Centre for International Trade and Development, School of International Studies, Jawaharlal Nehru University, New Mehrauli Road, New Delhi 110067, INDIA. Mobile: 91-9717675108. Email: sujata.eco@gmail.com

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1 Introduction

The focus of this study is to analyze the relation between intergenerational mobility (upward and downward mobility) and wage inequality (between skilled and unskilled workers) in a dynamic endogenous skill-biased growth framework where credit market is imperfect. Upward mobility captures the probability of becoming educated given that parent did not has formal education and downward mobility implies probability of not getting a formal education given that parent was educated. Katz and Murphy [1992], Berman et al. [Nov, 1998], Autor et al. [1998], Galor and Moav [2000] and Acemoglu [1998] show that wage dispersion between skilled and unskilled human capital widen in spite of an increment in the proportion of skilled human capital. Additionally, within skilled and unskilled human capital wage inequality is rising in the process of development. Ozdural [1993] and Björklund and Jäntti [1997] in cross-country study show that mobility. This implies that a more equal society leads to higher mobility. This study provides a theoretical set up which elucidate some empirical substantiation of high inequality associated with low mobility in the process of development.

Earlier literature on intergenerational mobility and income inequality consider exogenously given technology embodied with the production function. This study endogenously determines the technology level of the concerned economy through the twin channels of imitation and innovation, which is in line with Aghion et al. [2009], Vandenbussche et al. [2006] and Basu and Mehra [2014]. Technological progress is a dualistic phenomenon which uses human capital inputs differently at different stages of development (in terms of its distance to the world technology frontier).

Imperfect credit market has been captured through the level of bequest that parents left for their children. Rich parent can leave higher bequest than a poor parent. This may give a higher opportunity of becoming rich to a child of a rich parent than the child of a poor parent. Income of an individual depends not only on its own talent and income level but also on the education decision of his/ her parent and grandparent and so on.

This implies that, different generations are connected on the production side through the endogenous evolution of technology over time, and on the consumption side through income distributions of previous generations. Thus, in a dynamic setting, growth rate, aggregate income and intergenerational mobility of an economy are determined by the interaction of these two interrelated components. This study focuses on the convergence possibility of an economy to the world technology frontier. Additionally, in this part of research life time utility gaps within skilled human capital due to difference in cognitive ability at various stages of technological development have been examined. Moreover, an investigation has been made on the life time utility gaps within skilled as well as within unskilled human capital due to parental income differences. Finally, the impact of the different components of human capital on economic growth depending on its distance to frontier have been studied.

By looking at the literature on intergenerational mobility and inequality, it is found that without considering that the return to education changes over time, under imperfect capital market Becker and Tomes [1979] and Becker and Tomes [1986] and Loury [1981] show that a more equal society leads to higher

mobility and economic development. Becker and Tomes [1986] also shows that intergenerational mobility is smaller when endowments are transferred from parents to children. However, in these microeconomic studies dynasties act in isolation. Therefore, these analysis unable to capture the intensity of mobility for economies at different stages of development. By considering macroeconomic dynamics, Borjas [1992] empirically shows that the skills of the current generation depend not only on parental income but also on the average skills of the ethnic group in the parent's generation.

By considering the technological progress with perfect capital market, Galor and Tsiddon [1997] shows that a major technological invention amplify the return to ability and reduces the impact of the initial condition. This leads to an initial increment in inequality and consequently mobility. Over time, as technology becomes more accessible, both mobility and inequality decrease. There exists a positive relation between inequality and intergenerational mobility in the short run.

By combining the long run growth with the assumption of capital market imperfection, Owen and Weil [1998] and Maoz and Moav [1999] show that inequality and mobility move in opposite directions.¹ While Owen and Weil [1998] characterizes only the steady state condition, Maoz and Moav [1999] charts out the transitional dynamics of inequality, mobility and allocation of education along the growth path. However, both of these studies assume exogenously given technology and consider that growth is positively related with the proportion of skilled human capital in an economy. Therefor, along the growth path, an increment in the proportion of skilled human capital implies a reduction in the wage gap between these two factors (due to the diminishing return of factor productivity).² Therefore, due to a lower incentive effect and higher liquidity effect downward mobility increases and upward mobility decreases, that is, education becomes highly correlated with the ability.

Hassler et al. [2007] shows that differences in skill-biased technology or wage compression exhibits a positive relation between inequality and mobility.³ An economy with higher skill-biased technology or with lower wage compression leads to a higher inequality and higher mobility and vise versa. This holds under the assumption that poor parents not only have less ability to spend on children's education but also have a lower willingness to pay for it. Das [2007] shows that initial income differences persist even under convex technology and convex preferences. Empirical findings of Solon [1992], Solon [2002] and Lee and Solon [2009] show that intergenerational correlation in the long run income is relatively high for both son's and daughter's income. However, none of these studies have considered the possibility of endogenous Research and Development (R & D) based technological progress approach in the analysis of intergenerational mobility and inequality depending on the level of development of an economy.

This study also contributes to the existing literature on the importance of the different composition of human capital on growth depending on the distance of an economy from the world technology frontier.

¹The empirical support for this can be found in Andrews and Leigh [2009].

²In contrast Katz and Murphy [1992], Berman et al. [Nov, 1998], Autor et al. [1998], Galor and Moav [2000] and Acemoglu [2002] show that a higher growth trajectory leads to a higher inequality

³Wage compression implies that differences in wages between skilled and unskilled human capital is lower than the productivity level of them. A state with higher labor regulations or stronger labor union exhibits higher wage compression, that is, it leads to a more equal society.

Vandenbussche et al. [2006], Aghion et al. [2009] and Basu and Mehra [2014], theoretically, it is shown that when an economy is far away from the world technology frontier, imitation of technologies is the main engine of total factor productivity (TFP) growth. In comparison, as an economy bridges its gap from the world technology leader, the scope of imitation falls and the dependence on innovation activity rises. Technologically sufficiently advanced economies rely on innovation activity (constitute innovation-only regime) alone whereas technologically sufficiently backward economies perform only imitation activity (form the **imitation-only** regime). Intermediate economies perform both the activities (represent **imitation**innovation regime). The assumption that innovation is more skilled intensive that imitation is able to solve the puzzle posed by Krueger and Lindahl [2001] that education has positive and significant impact on growth only for the technologically backward economy but has negative and insignificant impact for the rich countries. ⁴ Vandenbussche et al. [2006] provides the explanation that unskilled human capital is more efficient in imitation activity and the scope of imitation is higher in a technologically backward economy. This line of argument is similar to Nelson and Phelps [1966]. On the contrary, Romer [1990] and Grossman and Helpman [1991] show that education favors the innovation of new technology (technologically advanced economy rely more on innovation activity) and skilled human capital is more efficient in it. By allowing inter-state migration, Aghion et al. [2009] also supports the theory postulated by Vandenbussche et al. [2006]. But both the earlier mentioned papers, that is, Vandenbussche et al. [2006] and Aghion et al. [2009] have assumed that there exists an exogenously given composition of human capital and it remains fixed irrespective of the distance of the economy to the frontier. With the assumption that perfect capital market prevails, Basu and Mehra [2014] endogenize the skill composition of an economy, based on an individual's decision to acquire education depending on the heterogeneous cognitive ability among individual's and shows the importance of skilled human capital in the diversified regime irrespective of its distance to frontier. Moreover, their study shows that skilled human capital is also growth enhancing in the innovation-only regime while unskilled human capital does the same job in the imitation-only regime. However, the above mentioned works do not consider that being educated is difficult for the individuals who are endowed with low resources. This work aims to extend this line of research to find out the significance of the different composition of human capital on growth for an economy depending on the level of development when individuals are credit constrained.

This work also interfaces with the existing literatures on convergence theory. Using the classical approach Barro et al. [1991], Barro et al. [1992] and Sala-i Martin [1994], Sala-i Martin [1996] show that there exists cross-sectional conditional β and σ convergence among the US states and for across different countries for different time periods. However, by using cointegration, Bernard and Durlauf [1995] shows very little evidence of the convergence of output. By criticizing the earlier methodology, Quah [1996a], Quah [1996b], Quah [1996c] Quah [1997], Quah [1999], Howitt and Mayer-Foulkes [2005] and Maasoumi et al. [2007] empirically show that income distribution is polarizing into twin peaks of rich and poor. With the assumption that an economy improves its technology level only through innovation, Howitt

⁴This nonlinear relation between human capital and economic growth is also supported by Durlauf and Johnson [1995] and Kalaitzidakis et al. [2001].

[2000] and by making the assumption that as technology level increases, innovation becomes more difficult, Howitt and Mayer-Foulkes [2005] theoretically demonstrate the possibility of club convergence. With nonparametric analysis Mayer-Foulkes [2002] shows that, in the long run, the world income converges in three steady states – semi-stagnation, semi-development and development, depending on whether countries have overcome barriers to human capital and technological innovation. Aghion and Howitt [2006], Di Maria and Stryszowski [2009] and Basu and Mehra [2014] show that without any distortion, such as migration, technology transfer from advanced to backward economy leads to absolute convergence in the long run. Next, the key findings of this analysis for imperfect credit market have been elaborated:

- 1. An endogenous technology evolution combined with credit market imperfection has been considered to understand the relation between mobilities and between skilled and unskilled group inequality depending on the distance of an economy from the world technology frontier. This specification is an improvement over Maoz and Moav [1999] and Owen and Weil [1998] in terms of the endogenous technology evolution. Similarly, unlike Basu and Mehra [2014], this study considers the heterogeneous agents not only in terms of cognitive ability but also in terms of the parental income status. Moreover, in contrast with above mentioned literature like Vandenbussche et al. [2006], Aghion et al. [2009] and Basu and Mehra [2014], this study includes that along with the advantage of backwardness there also exists disadvantage of backwardness as mentioned in Gerschenkron et al. [1962] and Howitt [2000].
- 2. A technologically sufficiently backward economy with low relative composition of skilled-unskilled human capital specializes in the imitation activity. This regime is known as the *imitation-only* regime. As the distance from the world technology frontier falls or the relative skill composition of the economy rises, it moves from the *imitation-only* regime to the *imitation-innovation* regime.⁵ In this diversified regime, the economy performs both imitation and innovation activities for technology enhancement. After that, the economy shifts from the *imitation-innovation* regime to *innovation-only* regime (performs only innovation activity) if either the economy becomes sufficiently technologically advanced or the relative proportion of skilled to unskilled human capital is significantly high.
- 3. The features of the labor market equilibrium conditions for the diversified regime as well as for the specialized regimes are derived. The assumption that innovation is relatively skilled human capital intensive implies that the reliance on unskilled human capital is relatively more for the imitation activity (under contant returns to scale (CRS) production structure). In equilibrium, this translates into a lower proportion of skilled human capital than unskilled human capital in the imitation-only regime. Moreover, given that the diminishing effect of imitation activity has a similar effect on both the factors, there exists a constant proportion of equilibrium composition of skilled and unskilled human capital in the imitation-only regime as an economy progresses to the world technology leader.

As an economy progresses further technologically, the scope of imitation falls. However, the opportunity for innovation rises. The assumption that innovation is skilled human capital intensive

⁵This regime is also known as diversified regime.

implies that the equilibrium proportion of skilled human capital rises and unskilled human capital falls as an economy progresses technologically in the diversified regime. Consequently, both skilled and unskilled human capital shift from the imitation to the innovation activity during the process in the diversified regime.

Finally, at the other extreme, an economy may depend on innovation activity only for future technology improvement. The assumption that innovation is more skilled intensive entails that the equilibrium proportion of skilled human capital is higher than unskilled human capital in the innovation-only regime. Further, there exists a constant composition of human capital in the innovation-only regime (since productivity of both the factors change at the same rate) as an economy moves toward the world technology frontier.

- 4. The analysis also examines the growth path of an economy depending on its distance from the frontier. In the imitation-only regime, as an economy progresses, the scope of imitation falls, and as an outcome of this the growth rate falls. There exists a declining trend displayed by the growth path of an economy in the imitation-only regime. In the diversified regime, there exists a U-shaped growth curve as the economy progresses. That is, growth rate initially falls only to rise later. However, the innovation-only regime exhibits a constant growth rate.
- 5. The long run dynamics of an economy are such that by implementing a growth enhancing education policy, the economy moves from the imitation-only regime to the imitation-innovation regime, and finally to the innovation-only regime. The assumption of zero population growth rate implies that in the long run, all the economies will converge to the world technology frontier and will grow at the same rate. *There exists absolute convergence among the economies.*
- 6. Further, the analysis focuses on the wage rate of skilled and unskilled workers and the wage inequality between skilled and unskilled workers in the three regimes. As an economy moves toward the world technology frontier, in the imitation-only (resp. innovation-only) regime, the marginal productivities of both skilled and unskilled human capital fall (resp. rise) leading to a reduction (resp. increment) in the wage rates of skilled and unskilled workers in the imitation-only (resp. innovation-only) regime. Moreover, a constant level of wage inequality is found to prevail between skilled and unskilled workers in both the imitation-only and innovation-only regimes.

In the imitation-innovation regime, as the economy progresses, the importance of innovation rises and that of imitation falls. Consequently, the marginal productivity of skilled human capital rises and that of unskilled human capital falls which leads to an *increment (resp. decrement) in the wage rate of skilled (resp. unskilled) workers in the diversified regime*. Additionally, *between skilled and unskilled groups, wage inequality rises as an economy moves to the frontier*. Owen and Weil [1998] and Maoz and Moav [1999] fail to capture this high growth which leads to higher inequality due to the consideration of production function embodied with exogenous technology evolution.

7. There exists a constant upward and downward mobility in the imitation-only and innovation-only

regimes (since a constant wage inequality prevails between skilled and unskilled workers). In the diversified regime, education becomes more correlated with the parental income and less related with the cognitive ability (as between group wage inequality rises) as the economy progresses. Both the upward and downward mobility decrease in the imitation-innovation regime. Endogenous technology improvement leads to a completely opposite findings of Owen and Weil [1998] and Maoz and Moav [1999] that along the growth path education becomes more correlated with the cognitive ability.

- 8. In the imitation-innovation regime as an economy progresses, the wage inequality between skilled and unskilled groups rises. This implies that the gap between the levels of bequest that individuals receive from their parent rises given the difference in the parental education status. Some of the individuals who are working as skilled (resp. unskilled) today, had skilled parents whereas others had unskilled parents. So, the levels of bequests received by today's skilled (resp. unskilled) workers vary according to their parental income status. Consequently, within skilled (resp. unskilled) human capital, the wealth gap rises depending on whether the parent was educated or not. This leads to an increase in the life time utility gap within both skilled and unskilled workers due to parental income differences in the diversified regime. There exists a constant level of life time utility gap within skilled and unskilled workers due to parental income differences in the imitation-only and innovation-only regimes. This is due to an unchanged wage inequality between skilled and unskilled workers in these two specialized regimes.
- 9. In the diversified regime, the relative skill composition of human capital rises as an economy moves toward the world technology frontier, which implies that individuals with relatively less cognitive ability now become skilled. As a result of this, wage inequality within skilled workers rises due to differences in cognitive ability. This, in turn, implies that the heterogeneity among skilled human capital rises. This leads to an increment in the life time utility gap within skilled workers due to differences in cognitive ability, irrespective of whether the parent was skilled or unskilled in the imitation-innovation regime. There exists a constant level of life time utility gap within skilled group due to differences in the cognitive ability in the imitation-only and innovation-only regimes (since there exists constant proportion of skilled and unskilled human capital in the specialized regimes).
- 10. By performing comparative static analysis, it is shown that skilled human capital is growth enhancing in the imitation-innovation and innovation-only regimes whereas unskilled human capital is growth maximizing in the specialized backward economy which is in line with Basu and Mehra [2014].

The paper is organized as follows. In Section 2, the basic structure of the model is discussed. Section 3 contains the key analytical results for a decentralized market economy which provides the main propositions of this study. Section 4 concludes and provides direction for future research. In what follows immediately, is the structure of the economy.

2 The Economic Environment

This section describes the structure of the economy. To begin with, the production structure of the economy, which resembles the Aghion and Howitt [1992] creative destruction model with quality ladders is elaborated. Next, the focus is placed on the structure of the dynamics of technology improvement of different economies depending on their distance from the world technology frontier. This structure is in line with Vandenbussche et al. [2006], where the technology enhancement depends on both the imitation and the innovation activities. Followed by this, the consumption side of the economy is demonstrated. Individuals care for their children and the capital market is imperfect, as in Maoz and Moav [1999]. First, the discussion starts with the production side of the economy.

2.1 Production

There are a finite number of small open economies. Similar to Aghion and Howitt [1992], in each economy there is an entrepreneur, who is engaged in the production of a final output in a perfectly competitive market. There are a continuum of mass one of intermediate input producers, who produce the monopoly output, and invest their monopoly profit in the R & D activity. In each time period t, with certainty one intermediate producer invents the highest available technology/ blueprint for each of the intermediate goods and after that he/ she produces the good with this technology/ blueprint. In the next period, he/ she leaves the market and a new intermediate input producer arrives. There exists free entry and exit in the R & D sector. That is, the R & D sector is also perfectly competitive. The price of the final good is normalized to one. All modeling is done using discrete time interval.

The final output is produced competitively, by using land and a continuum of mass one of intermediate inputs. Cobb-Douglas production function of the following form is considered:

$$Y_{\rm t+1} = l_{\rm t+1}^{1-\alpha} \int_0^1 A_{\rm i,t+1}^{1-\alpha} x_{\rm i,t+1}^{\alpha} di, \quad 0 < \alpha < 1,$$

where *i* denotes the *i*th intermediate sector, Y_{t+1} is the final output in period t+1, l_{t+1} is the total amount of land, $A_{i,t+1}$ is the technology level in sector *i* in period t+1 and $x_{i,t+1}$ is the amount of intermediate input used in sector *i* in period t+1. For simplicity, the total supply of land is normalized to one.

Final good sector produces under perfect competition. Therefore, the price of each of the intermediate inputs i is equal to its marginal product, that is,

$$p_{i,t+1} = \frac{\partial Y_{t+1}}{\partial x_{i,t+1}} = \alpha A_{i,t+1}^{1-\alpha} x_{i,t+1}^{\alpha-1},$$

where $p_{i,t+1}$ denotes the price of the intermediate input in sector *i* in period t + 1. Each intermediate input producer chooses output by maximizing the present discounted value of future profits. Since each intermediate input producer works for one time period only, the optimization exercise is equivalent to maximizing the profit period by period. The total revenue of the intermediate producer is the product of the intermediate inputs sold and its price, that is, $p_{i,t+1}x_{i,t+1}$. Total profit is calculated by subtracting cost from revenue. Once the new technology is invented, it is assumed that one unit of final good is required to produce one unit of intermediate input. Given the assumption that the price of the final output is normalized to one, the total cost of intermediate input producer to produce the inputs is $x_{i,t+1}$. Monopolist chooses $x_{i,t+1}$ by solving

$$\max_{x_{i,t+1}} (p_{i,t+1} x_{i,t+1} - x_{i,t+1}).$$

Accordingly, the monopolist produces the following amount of the intermediate good in sector i in period t + 1:

$$x_{\mathbf{i},\mathbf{t}+1} = \alpha^{\frac{2}{1-\alpha}} A_{\mathbf{i},\mathbf{t}+1}$$

The profit of the intermediate input producer is:

$$\pi_{i,t+1} = (p_{i,t+1} - 1)x_{i,t+1} = \left(\frac{1}{\alpha} - 1\right)\alpha^{\frac{2}{1-\alpha}}A_{i,t+1} = \delta A_{i,t+1},\tag{1}$$

where $\delta = \left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{2}{1-\alpha}}$.

Note that both the equilibrium level of production and the profit of the intermediate input producer in sector i in period t + 1 are linearly dependent on the local/ national technology level in sector i in that period. Both, the technology adjusted intermediate inputs and the profit are the same for all the sectors in every period.

2.2 Dynamics of Productivity

Technological progress depends not only on the innovation upon local/ national technology level but also on the imitation of technology from the world technology frontier. This is similar to Benhabib and Spiegel [1994] and Acemoglu et al. [2006]. However, in both these papers, technology improvement depends on the total stock of human capital and not on its composition. This implies that whether skilled or unskilled human capital are engaged in imitation or innovation activities, does not have any impact on technology enhancement. This is a rather restrictive assumption. So, by improving upon this, a specification as in Vandenbussche et al. [2006] and Aghion et al. [2009] is considered, where imitation and innovation activities require both skilled and unskilled human capital, but with differing intensity of use for each type of activity. It is assumed that innovation is relatively skilled human capital intensive. In a CRS frame work, it implies that imitation is unskilled human capital intensive. This entails that a technologically backward (resp. advanced) economy specializes in imitation (resp. innovation) activity. The intermediate economies perform both the activities.⁶

2.2.1 Imitation-Only Regime

The technology improvement specification of an economy which is in the imitation-only regime is:

$$A_{i,t+1} = A_{i,t} + \lambda \widetilde{U}^{\sigma}_{i,t+1} \widetilde{S}^{1-\sigma}_{i,t+1} \frac{1}{\overline{A}_{t+1}} (\overline{A}_t - A_t), \quad \lambda > 0,$$

$$(2)$$

^{6}Lemma 3 in Subsection 3.4 on page 24 shows that this is true in equilibrium.

where, $\widetilde{U}_{i,t+1}$ and $\widetilde{S}_{i,t+1}$ respectively measure the levels of unskilled and skilled human capital in the imitation-only regime, σ is the elasticity of unskilled human capital in the imitation activity and λ captures the efficiency of the overall technology improvement. A_t measures the aggregate technology level of the concerned economy in period t, where, $A_t = \int_0^1 A_{it} di$. \overline{A}_t measures the aggregate technology level of the world leader, such that, $\overline{A}_t = \int_0^1 \overline{A}_{it} di$. $\frac{(\overline{A}_t - A_t)}{A_{t+1}}$ captures the scope of imitation, that is, the gap of the concerned economy's technology level from the world leader. Along with the advantage of backwardness, there exists a disadvantage of backwardness, as mentioned by Gerschenkron et al. [1962]. This is captured by the scope of imitation being divided by its targeted world technology level, that is, \overline{A}_{t+1} to imply an inverse relation to technology to the world technology level. More advanced the world leader, more difficult it is to imitate for a backward economy.

2.2.2 Imitation-Innovation Regime

The technology improvement pattern for an economy which is in the imitation-innovation regime is postulated as:

$$A_{i,t+1} = A_{i,t} + \lambda \left[u^{\sigma}_{mi,t+1} s^{1-\sigma}_{mi,t+1} \frac{1}{\overline{A}_{t+1}} (\overline{A}_t - A_t) + \gamma \ u^{\phi}_{ni,t+1} s^{1-\phi}_{ni,t+1} A_t \right], \quad \gamma > 0,$$
(3)

where, $u_{\text{mi,t+1}}$ and $s_{\text{mi,t+1}}$ respectively denote the amounts of unskilled and skilled human capital engaged in the imitation activity in the diversified regime, $u_{\text{ni,t+1}}$ and $s_{\text{ni,t+1}}$ respectively measure the amounts of unskilled and skilled human capital employed in the innovation activity in the diversified regime, ϕ is the elasticity of unskilled human capital in the innovation activity and γ measures the relative efficiency of innovation as compared to imitation. This implies that in the diversified regime an economy can improve its technology level through two channels: imitation captured by $(\overline{A}_t - A_t)$ and innovation reflected in the level of A_t .

2.2.3 Innovation-Only Regime

In the innovation-only regime, an economy is so advanced that technology enhancement depends on innovation only – the efficiency by which skilled and unskilled human capital innovate determines the next period technology level.⁷ The technology evolution process for this specialized advanced economy is characterized by:

$$A_{i,t+1} = A_{i,t} + \lambda \gamma \widehat{U}^{\phi}_{i,t+1} \widehat{S}^{1-\phi}_{i,t+1} A_t, \qquad (4)$$

where, $\hat{U}_{i,t+1}$ and $\hat{S}_{i,t+1}$ respectively measure the levels of unskilled and skilled human capital in the innovation-only regime.

To satisfy the basic assumption that innovation is relatively skilled human capital intensive than imitation, the following specific assumption is made:

⁷Here efficiency or productivity of skilled and unskilled human capital are measured in terms of the elasticity of skilled and unskilled human capital in the imitation and innovation activities.

A1. The elasticity of skilled human capital is higher in the innovation activity than in the imitation activity, that is, $\sigma > \phi$. In the same vein, under the imitation-only regime, imitation is unskilled human capital intensive, implying that $\sigma > \frac{1}{2}$ and in the innovation-only regime, innovation is skilled human capital intensive, such that $\phi < \frac{1}{2}$.⁸

A2. World technology frontier is growing at a constant exogenous rate \bar{g} .

2.3 Consumption Side

An individual lives for two time periods in an overlapping generations model. He/ she has a log-linear utility function. Utility depends on individual's consumption in both the periods and the level of bequest that he/ she leaves for his/ her child. In the first period of the life, an individual takes a decision on whether to opt for education or not. In the second period, depending on the education decision taken, he/ she works as skilled/ unskilled worker. Like Maoz and Moav [1999], a complete absence of capital market is assumed, so that individuals cannot borrow or lend. In other words, income and expenditure in any two periods are independent. An individual spends the bequest received on the first period consumption and education (if it opts for it) and allocates the second period income on own consumption and leaves a bequest for his/ her child. Individuals vary in their cognitive ability, captured by the parameter θ , which is uniformly distributed over the interval [0, 1]. The cost of education is negatively related to the individual's cognitive ability and positively with the wage rate of unskilled worker. This is considered as the opportunity cost of an individual to become skilled, that is,

$$E(\theta, A_{t-1}) = \frac{Hw_{\rm ut}}{\theta},\tag{5}$$

where $E(\theta, A_{t-1})$ captures the cost of education of an individual with θ cognitive ability and H is any positive constant and w_{u_t} is the wage rate of unskilled workers in period t. Both skilled and unskilled workers maximize their lifetime utility subject to the budget constraint. Each individual maximizes the following lifetime utility function:

$$\mathbb{W}_k = c_{\mathbf{k},\mathbf{t},\mathbf{t}} \sqrt{c_{\mathbf{k},\mathbf{t},\mathbf{t}+1} x_{\mathbf{k},\mathbf{t},\mathbf{t}+1}},\tag{6}$$

where k = s, u. s and u respectively denote skilled and unskilled workers. \mathbb{W}_k measures the lifetime utility of the k^{th} individual, $c_{k,t,t}$ is the consumption level of the k^{th} individual in period t who is born in period $t, c_{k,t,t+1}$ is the consumption level of k^{th} individual in period (t + 1) who is born in period t, and $x_{k,t,t+1}$ is the level of bequest that k^{th} individual who is born in period t leaves for his/ her child in period (t + 1). The budget constraint of skilled worker who is born in period t is given as:

$$\begin{split} c_{\mathrm{s,t,t}} + \frac{H w_{\mathrm{ut}}}{\theta} &= x_{\mathrm{t,t}}; \\ c_{\mathrm{s,t,t+1}} + x_{\mathrm{s,t,t+1}} &= w_{\mathrm{s,t+1}}, \end{split}$$

⁸In the diversified regime this analysis does not require any assumption on the absolute intensity of skilled or unskilled human capital in the imitation or innovation activities. Hence, these parametric restrictions pertain only to the specialized economies.

where $w_{s,t+1}$ measures the wage rate of skilled worker in period (t+1), $x_{t,t}$ is the level of bequest that an individual received from his/ her parent. It depends on whether his/ her parent was skilled or unskilled worker. The budget constraint of unskilled worker who is born in period t is

$$c_{u,t,t} = x_{t,t};$$

 $c_{u,t,t+1} + x_{u,t,t+1} = w_{u,t+1}.$

Perfectly competitive labor market is assumed. Individuals have perfect foresight. Total population is normalized to one. There is no population growth. Each parent has one child. At the end of the t^{th} generation, a new $(t+1)^{th}$ generation appears.

The interaction of production and consumption activities determines the equilibrium composition of human capital. This, in turn, determines the allocation of skilled and unskilled human capital between the imitation and innovation activities that ascertains the overall technology improvement. Consequently, this determines the growth path, convergence condition, wage, inequality and intergenerational mobility paths of the economy as the time progresses.

3 Key Analytical Results

In this section key analytical findings of this research have been derived. First, the labor supply curve is obtained. Next, the focus of the analysis is on the imitation-only regime followed by that on the innovation-only regime and finally on the imitation-innovation regime. Under each case, the equilibrium composition of human capital, growth curve and the wage paths of both the factors have been characterized. Furthermore, the long run steady state condition has been examined. In addition, the relation between intergenerational mobility (upward and downward mobility) and the wage inequality of an economy depending on its distance to frontier have been illustrated. Finally, comparative dynamics are worked out to understand the importance of the composition of human capital at different stages of development.

3.1 Labor Supply

A detailed analysis has been made to determine the labor supply curve of the economy. As already mentioned, income and consumption in two periods are not interrelated (since credit market does not exist.). Thus, the second period utility function subject to the second period budget constraint has been maximized. A log-linear utility function ensures that an individual spends his/ her income equally on second period consumption and bequest, that is, $c_{k,t,t+1} = x_{k,t,t+1} = \frac{w_{k,t+1}}{2}$.⁹ An individual opts for education if his/ her lifetime income as skilled worker is greater than unskilled worker, specifically,¹⁰

⁹Detailed mathematical derivations are provided in the cluster of eqs. (??) in Appendix A on page ??.

¹⁰Detailed mathematical derivations are provided in the cluster of eqs. (??) in Appendix A on page ??.

$$\mathbb{W}_{s} \geq \mathbb{W}_{u}
\Rightarrow \quad \theta_{t+1} \geq \frac{Hw_{ut}}{x_{t,t} \left[1 - \frac{w_{u,t+1}}{w_{s,t+1}}\right]}.$$
(7)

An individual avails the education option if his/ her cognitive ability is higher than a threshold level as mentioned in eq. (7). As expected, this depends on the future wage gap between skilled and unskilled human capital (incentive effect), level of bequest that an individual received from his/ her parent (wealth effect) and also on the cost of education (opportunity cost effect). If an individual's parent was skilled, he/she receives a higher bequest (that is, $\frac{w_{st}}{2}$) than an individual whose parent was unskilled (that is, $\frac{w_{ut}}{2}$). Therefore, the cutoff level of cognitive ability above which an individual goes for education depends on whether his/ her parent was educated or not. Thus, we have,

$$\theta_{t+1}^{u} = \frac{2H}{1 - \frac{w_{u,t+1}}{w_{s,t+1}}} \quad \text{and} \quad \theta_{t+1}^{s} = \frac{2H}{1 - \frac{w_{u,t}}{x_{s,t}}},\tag{8}$$

where θ_{t+1}^u and θ_{t+1}^s respectively measure the cut off cognitive ability above which an individual goes for education if his/ her parent was unskilled and skilled. Note that $\theta_{t+1}^s < \theta_{t+1}^u$. It implies that the child of an educated parent has higher opportunity of acquiring education than the child of an uneducated parent. So, education decision is not only correlated with the cognitive ability of an individual but is also related to the parental education decision and income. This finding is in line with Maoz and Moav [1999]. Therefore, the proportion of unskilled (resp. skilled) human capital in period (t + 1) is a weighted average of proportion of uneducated (resp. educated) individuals in period (t+1) having educated parent in period t and the proportion of uneducated (resp. educated) individuals in period (t+1) having uneducated parent in period t.¹¹ The proportions of unskilled and skilled human capital in period (t + 1) are respectively:

$$U_{t+1} = \theta_{t+1}^{u} U_{t} + \theta_{t+1}^{s} S_{t} = \frac{2H \left[U_{t} + \frac{x_{u,t}}{x_{s,t}} S_{t} \right]}{1 - \frac{w_{u,t+1}}{w_{s,t+1}}};$$

$$S_{t+1} = 1 - U_{t+1}.$$
 (9)

The proportion of unskilled (resp. skilled) human capital in period (t + 1) depends on the composition of human capital in period t and also on the expected future wage gap of skilled and unskilled workers. So, there is a trade off between *history vs. expectation*, as mentioned in Krugman [1991]. Historically if high wage inequality prevails in the economy (that is, wealth effect is significant), it leads to a high proportion of unskilled human capital. Similarly, if previous period's relative composition of skilled human capital is low, it also implies a high proportion of unskilled human capital in the next period. Both of these two factors capture the history effect. However, if expected wage rate of skilled human capital is higher than the unskilled one (incentive effect), it leads to a lower proportion of unskilled human capital. Thus, it is important to understand the countering effects of history vs. expectation.

¹¹Educated individuals constitute the skilled set. So, this part of research use these two terminologies as synonyms.

3.2 Imitation-Only Regime

In this subsection the economies which are in the imitation-only regime are characterized. First, the demand levels for skilled and unskilled workers are derived. Next, the equilibrium proportions of skilled and unskilled workers are ascertained. Finally, the growth path of an economy and the wage paths of skilled and unskilled workers are determined.

3.2.1 Demand for Skilled and Unskilled Human Capital

The demand curve of skilled and unskilled human capital in the imitation-only regime can be derived as follows. Since an intermediate input producer operates the production process for one period only, he/ she maximizes current profit net of labor costs. From eqs. (1) and (2), the profit maximizing programme of the intermediate input producer is:

$$\max_{\widetilde{\mathcal{U}}_{i,t+1},\widetilde{S}_{i,t+1},\delta} \delta A_{it} + \lambda \delta \widetilde{\mathcal{U}}_{i,t+1}^{\sigma} \widetilde{S}_{i,t+1}^{1-\sigma} \frac{1}{\overline{A}_{t+1}} (\overline{A}_{t} - A_{t}) - \left[w_{u,t+1} \widetilde{\mathcal{U}}_{i,t+1} + w_{s,t+1} \widetilde{S}_{i,t+1} \right],$$
(10)

where $\widetilde{w}_{i,t+1} = \left[w_{u,t+1}\widetilde{U}_{i,t+1} + w_{s,t+1}\widetilde{S}_{i,t+1}\right]$ measures the total labor cost of R & D activity in the imitationonly regime. From eq. (10), the first order conditions of the maximization exercise of R & D activity have been derived in the imitation-only regime to be:

$$\frac{\partial \mathbb{L}_{1,t+1}^{M}}{\partial \widetilde{U}_{i,t+1}} = \lambda \delta_{1} \sigma \widetilde{U}_{i,t+1}^{\sigma-1} \widetilde{S}_{i,t+1}^{1-\sigma} \frac{1}{(1+\bar{g})} (1-a_{t}) - w_{u,t+1} = 0; \\
\frac{\partial \mathbb{L}_{1,t+1}^{M}}{\partial \widetilde{S}_{i,t+1}} = \lambda \delta_{1} (1-\sigma) \widetilde{U}_{i,t+1}^{\sigma} \widetilde{S}_{i,t+1}^{-\sigma} \frac{1}{(1+\bar{g})} (1-a_{t}) - w_{s,t+1} = 0,$$
(11)

where $a_t = \frac{A_t}{\overline{A}_t}$ measures the inverse distance of an economy from the world technology frontier. In this research this is termed as *distance to frontier*.

From the first order conditions expressed in the cluster of eqs. represented as (11), the relative demand curve for skilled and unskilled human capital in the imitation-only regime can be expressed as:

$$\frac{w_{\mathrm{u,t+1}}}{w_{\mathrm{s,t+1}}} = \frac{\sigma}{(1-\sigma)} \frac{\widetilde{S}_{\mathrm{i,t+1}}}{\widetilde{U}_{\mathrm{i,t+1}}}.$$
(12)

Eq. (12) says that the equilibrium relative wage rate of skilled worker decreases as the relative demand for skilled human capital rises. So, the relative demand curve is negatively sloped in relative wages.

3.2.2 Equilibrium

Next, the equilibrium proportion of skilled and unskilled human capital in the imitation-only regime has been analyzed. A perfectly competitive labor market ensures that at a competitive wage rate, labor demand equates labor supply. From eqs. (9) and (12), the proportion of unskilled human capital in the imitation-only regime is:¹²

$$\widetilde{U}_{t+1} = \sigma + 2H \left(1 - \sigma\right) \left[\widetilde{U}_{t} + \frac{x_{u,t}}{x_{s,t}}\widetilde{S}_{t}\right].$$
(13)

¹²Detailed mathematical expressions are provided in the cluster of eqs. (??) in Appendix A on page ??.



Figure 1: Imitation-Only Regime - Composition of Skilled and Unskilled Human Capital and Growth Rate

Eq. (13), yields the proportion of skilled human capital as:¹³

$$\widetilde{S}_{t+1} = 1 - \widetilde{U}_{t+1} = (1 - \sigma) \left[1 - 2H \left(\widetilde{U}_t + \frac{x_{u,t}}{x_{s,t}} \widetilde{S}_t \right) \right].$$
(14)

Eq. (13), exhibits that there exists a positive proportion of unskilled human capital in the imitation-only regime. To ensure the essentiality of skilled input, the following condition is required:

$$\widetilde{S}_{t+1} > 0 \quad \Rightarrow \quad H < \frac{1}{2 \left[\widetilde{U}_t + \frac{x_{u,t}}{x_{s,t}} \widetilde{S}_t \right]}.$$
(15)

Condition in eq. (15) is not bounded.¹⁴ Now, the question is: how do the equilibrium proportions of skilled or unskilled human capital change as an economy bridges the gap from the frontier. This depends on the last period's wage inequality and the earlier period's proportion of skilled and unskilled human capital. The last period's equilibrium values depend on the last to last period and the process continues. Thus, the labor market equilibrium condition of the initial period determines today's outcome. These are history dependent in the imitation-only regime, which renders them mathematically tractable. Therefore, simulation technique has been used. The arbitrary parameter values taken for numerical simulations are:

Parameters	λ	γ	δ	σ	ϕ	Η	\bar{g}	$\overline{A}(1)$
Values	0.4	0.1	0.6	0.6	0.15	0.1	0.02	10

 Table 1: Parameter Values for Numerical Simulation

¹³Detailed mathematical derivations are provided in the cluster of eqs. (??) in Appendix A on page ??.

¹⁴By using **Lemma 3** in Subsection 3.4 on page 24, unboundedness of this condition has been shown in the cluster of eqs. (??) in Appendix A on page ??.

The specific parameter values that represent the initial conditions for the imitation-only regime are the following:

Parameters	a(1)	U(1)
Values	0.001	0.8

Table 2: Specific Parameter Values for Numerical Simulation in the Imitation-only Regime

All of these parameter values satisfy the regularity condition mentioned in Lemma 3 in Subsection 3.4 on page 24. These specific parameter values represent the case where the economy is sufficiently backward and has a relatively high composition of unskilled human capital.¹⁵ Given that the initial parameter values are chosen as to satisfy the regularity conditions, there exists an initial change in the equilibrium proportion of unskilled human capital. Given diminishing return to the imitation activity (since the scope of imitation falls) as an economy progresses, the marginal productivities of both skilled and unskilled human capital fall. However, this negative effect is similar for both the factor inputs. Thus, there exists a constant composition of human capital (as is shown in Fig. 1a on page 14). By A1, imitation is unskilled human capital intensive. This implies that the equilibrium level of unskilled human capital is higher than skilled human capital in the imitation-only regime (as is illustrated in Fig. 1a on page 14). This implies that the introduction of the imperfect capital market does not change the labor market equilibrium finding of the imitation-only regime of Basu and Mehra [2014].

3.2.3 Growth Rate

The growth rate of an economy in the imitation-only regime has been characterized. From eq. (2), we get,¹⁶

$$\widetilde{g}_{t+1} = \int_{0}^{1} \frac{A_{i,t+1} - A_{i,t}}{A_{it}} \mathrm{di} = \frac{\lambda}{(1+\bar{g})\overline{A}_{t}} \widetilde{U}_{t+1}^{\sigma} \widetilde{S}_{t+1}^{(1-\sigma)} \frac{(1-a_{t})}{a_{t}},$$

where, \tilde{g}_{t+1} measures the growth rate of an economy in the specialized backward regime in period t + 1. The growth rate of the economy depends on the composition of skilled and unskilled human capital and its distance to frontier. This implies that the growth path is history dependent (since the composition of human capital is history dependent). Therefore, an analytical solution is hard to get. So, again numerical simulation has to be used. As the relative gap of an economy from the world technology frontier shrinkens, the scope of imitation falls. Consequently, the increment to technology is lower. This leads to a lower growth rate as an economy progresses in the specialized backward regime (as is shown in Fig. 1b on page 14). This result is also in line with Basu and Mehra [2014].

¹⁵Lemma 3 postulates these characteristics for the imitation-only regime.

¹⁶Detailed mathematical derivations are provided in the cluster of eqs. (??) in Appendix A on page ??.



Figure 2: Imitation-Only Regime – Wage and Inequality Paths of Skilled-Unskilled Human Capital

3.2.4 Wage Rate

The wage paths of skilled and unskilled workers in the imitation-only regime are illustrated. Furthermore, the focus is on the analysis of the relative wage gap between skilled and unskilled workers. From the cluster of eqs. (11) and (13), the following expressions are obtained:

$$\widetilde{w}_{u,t+1} = \lambda \delta \frac{\sigma(1-\sigma)^{(1-\sigma)}}{(1+\bar{g})} \frac{\left[1-2H\left(\widetilde{S}_{t}+\frac{x_{s,t}}{x_{u,t}}\widetilde{U}_{t}\right)\right]^{(1-\sigma)}}{\left[\sigma+2H\left(1-\sigma\right)\left(\widetilde{S}_{t}+\frac{x_{s,t}}{x_{u,t}}\widetilde{U}_{t}\right)\right]^{(1-\sigma)}}(1-a_{t});$$

$$\widetilde{w}_{s,t+1} = \lambda \delta \frac{(1-\sigma)^{(1-\sigma)}}{(1+\bar{g})} \frac{\left[1-2H\left(\widetilde{S}_{t}+\frac{x_{s,t}}{x_{u,t}}\widetilde{U}_{t}\right)\right]^{-\sigma}}{\left[\sigma+2H\left(1-\sigma\right)\left(\widetilde{S}_{t}+\frac{x_{s,t}}{x_{u,t}}\widetilde{U}_{t}\right)\right]^{-\sigma}}(1-a_{t}).$$
(16)

Similar to the equilibrium proportion of skilled and unskilled human capital, the wage rates of the different composition of workers are also history dependent (as is visible from the cluster of eqs. (16)). Therefore, analytical solutions are not feasible, and hence numerical simulation has to be resorted to. As an economy progresses, the scope of imitation falls; consequently, the marginal productivity of both skilled and unskilled workers fall and so do the wage paths of both the factors (also is depicted in Fig. 2a on page 16). However, this diminishing effect is identical for both the factors. Consequently, there exists a constant level of wage inequality between skilled and unskilled workers in the imitation-only regime (Fig. 2b on page 16 also supports this). These findings are also analogues with Basu and Mehra [2014].

3.3 Innovation-Only Regime

Next, the chapter focuses on an economy which is in the innovation-only regime. This subsection derives the equilibrium proportion of skilled and unskilled human capital. Consequently, this determines the growth rate of the economy. Furthermore, the wage paths of the economy in the innovation-only regime are derived.

3.3.1 Demand for Skilled and Unskilled Human Capital

The demand curves for skilled and unskilled workers are derived. From eqs. (1) and (4), the maximization problem of the R & D activity of the intermediate input producers in the innovation-only regime will be:

$$\max_{\widehat{U}_{i,t+1},\widehat{S}_{i,t+1},} \delta A_{i,t} + \lambda \gamma \delta_1 \widehat{U}_{i,t+1}^{\phi} \widehat{S}_{i,t+1}^{1-\phi} A_t - \left[w_{u,t+1} \widehat{U}_{i,t+1} + w_{s,t+1} \widehat{S}_{i,t+1} \right],$$

where $\widehat{w}_{i,t+1} = \left[w_{u,t+1} \widehat{U}_{i,t+1} + w_{s,t+1} \widehat{S}_{i,t+1} \right]$ measures the cost associated with the R & D activity of an intermediate input producer who is in the innovation-only regime.

The first-order conditions associated with this maximization exercise are:

$$\frac{\partial \mathbb{L}_{1,t+1}^{N}}{\partial \widehat{U}_{i,t+1}} = \lambda \delta \gamma \phi \widehat{U}_{i,t+1}^{\phi-1} \widehat{S}_{i,t+1}^{1-\phi} A_{t} - w_{u,t+1} = 0;$$

$$\frac{\partial \mathbb{L}_{1,t+1}^{N}}{\partial \widehat{S}_{i,t+1}} = \lambda \gamma \delta (1-\phi) \widehat{U}_{i,t+1}^{\sigma} \widehat{S}_{i,t+1}^{-\phi} A_{t} - w_{s,t+1} = 0.$$
(17)

From the cluster of eqs. (17), the relative demand for skilled-unskilled human capital in the innovation-only regime is derived as:

$$\frac{w_{\rm u,t+1}}{w_{\rm s,t+1}} = \frac{\phi}{(1-\phi)} \frac{\widehat{S}_{\rm i,t+1}}{\widehat{U}_{\rm i,t+1}}.$$
(18)

The implication is that a negatively sloped demand curve is obtained. As the relative wage rate of skilled worker rises the relative demand for skilled human capital falls.

3.3.2 Equilibrium

By equating the demand and supply curves of human capital, the equilibrium proportions of skilled and unskilled human capital in the innovation-only regime have been derived from eqs. (9) and (18) as:¹⁷

$$\widehat{U}_{t+1} = \phi + 2H \ (1 - \phi) \left[\widehat{U}_{t} + \frac{x_{u,t}}{x_{s,t}} \widehat{S}_{t} \right];$$

$$\widehat{S}_{t+1} = (1 - \phi) \left[1 - 2H \ \left(\widehat{U}_{t} + \frac{x_{u,t}}{x_{s,t}} \widehat{S}_{t} \right) \right].$$
(19)

The equilibrium proportion of unskilled human capital is always positive. The regularity condition for the positive stock of skilled human capital is the following:

Now,
$$\widehat{S}_{t+1} > 0 \Rightarrow H < \frac{1}{2\left[\widehat{U}_t + \frac{x_{u,t}}{x_{s,t}}\widehat{S}_t\right]}.$$
 (20)

Condition in eq. (20) is not bounded.¹⁸ Similar to the imitation-only regime, here also the stocks of skilled and unskilled human capital in period (t+1) are history dependent. Thus, the behavior of the composition

¹⁷Detailed mathematical derivations are provided in the cluster of eqs. ??-?? in Appendix A on page ??.

¹⁸By using **Lemma 3** in Subsection 3.4 on page 24, unboundedness of this condition has been shown in the cluster of eqs. (??) in Appendix A on page ??.



Figure 3: Innovation-Only Regime – Composition of Skilled and Unskilled Human Capital and Growth Rate

of human capital has been characterized by applying numerical simulations. The specific parametric values for the innovation-only regime are the following:

Parameters	a(1)	U(1)
Values	0.6	0.3

Table 3: Specific Parameter Values for Numerical Simulation in the Innovation-only Regime

These capture the feature of an economy which is in the advanced specialized regime. These imply that the economy is sufficiently advanced and has a high enough proportion of skilled to unskilled ratio. These also satisfy all the regularity conditions imposed by the model as is illustrated in eq. (20) and also in **Lemma 3** in Subsection 3.4 on page 24. By **A1**, innovation is skilled intensive. Therefore, the proportion of skilled human capital is higher than unskilled human capital in the innovation-only regime (as is clear from Fig. 3a on page 18). Moreover, as an economy progresses, the marginal productivities of both skilled and unskilled human capital rise. However, this increment is same for both the factors. Therefore, there exists a constant proportion of skilled and unskilled human capital. These findings of the labor market equilibrium condition of the innovation-only regime are similar with Basu and Mehra [2014] where capital market is perfect.

3.3.3 Growth Rate

The growth rate of an economy in the innovation-only regime denoted as \hat{g}_{t+1} has been derived by using eq. (4) to be:

$$\widehat{g}_{t+1} = \lambda \gamma \widehat{U}_{t+1}^{\phi} \widehat{S}_{t+1}^{(1-\phi)}.$$
(21)



Figure 4: Innovation-Only Regime – Wage and Inequality Paths of Skilled and Unskilled Human Capital

The growth rate depends on the composition of human capital, such that, this too turns out to be history dependent. From eq. (21), it is clear that (given a fixed proportion of skilled and unskilled human capital) there exists a constant level of growth rate in the innovation-only regime (as is shown in Fig. 3b on page 18) which is analogues with the findings of Basu and Mehra [2014].

3.3.4 Wage Rate

Next, wage paths of skilled and unskilled workers are discussed. From the cluster of eqs. in (17), we get,

$$\widehat{w}_{u,t+1} = \lambda \delta \phi \widehat{U}_{t+1}^{\phi-1} \widehat{S}_{t+1}^{1-\phi} a_t \overline{A}_t;$$

$$\widehat{w}_{s,t+1} = \lambda \delta (1-\phi) \widehat{U}_{t+1}^{\phi} \widehat{S}_{t+1}^{-\sigma} a_t \overline{A}_t.$$
(22)

These expressions imply that as an economy progresses, the technology level rises, which entails an increment in efficiency of the innovation activity. This raises the marginal productivities of both skilled and unskilled workers and consequently, increase the wage rate of both types of workers. This is also depicted in Fig. 4a on page 19. However, these increment are the same for both the factors. This implies that after the initial adjustment (due to the choice of initial values) there exists a constant level of between group wage inequality in the innovation-only regime. Numerical simulation also exhibits a similar result as can be seen in Fig. 4b on page 19. These findings are in line with Basu and Mehra [2014].

3.4 Imitation-Innovation Regime

This subsection first derives the demand curve for both skilled and unskilled human capital in the imitation and in the innovation activities. Subsequently, the equilibrium allocation of both types of human capital are ascertained. Finally, the growth rate and wage paths of skilled and unskilled workers for an economy which is in the diversified regime have been characterized.

3.4.1 Demand for Skilled and Unskilled Human Capital

From eqs. (1) and (3), the maximization problem of the R & D producer in the diversified regime will be:

$$\max_{u_{\mathrm{mi},\mathrm{t+1}},u_{\mathrm{ni},\mathrm{t+1}},s_{\mathrm{mi},\mathrm{t+1}},s_{\mathrm{ni},\mathrm{t+1}}} \lambda \delta \left[u_{\mathrm{mi},\mathrm{t+1}}^{\sigma} s_{\mathrm{mi},\mathrm{t+1}}^{1-\sigma} \frac{1}{\overline{A}_{\mathrm{t+1}}} (\overline{A}_{\mathrm{t}} - A_{\mathrm{t}}) + \gamma u_{\mathrm{ni},\mathrm{t+1}}^{\phi} s_{\mathrm{ni},\mathrm{t+1}}^{1-\phi} A_{\mathrm{t}} \right] - \left[w_{\mathrm{u},\mathrm{t+1}} (u_{\mathrm{mi},\mathrm{t+1}} + u_{\mathrm{ni},\mathrm{t+1}}) + w_{\mathrm{s},\mathrm{t+1}} (s_{\mathrm{mi},\mathrm{t+1}} + s_{\mathrm{ni},\mathrm{t+1}}) \right],$$
(23)

where $w_{i,t+1} = [w_{u,t+1}(u_{mi,t+1} + u_{ni,t+1}) + w_{s,t+1}(s_{mi,t+1} + s_{ni,t+1})]$ measures the labor cost of R & D activity in the diversified regime. The first-order maximizing conditions in the R & D sector are obtained from eq. (23) for the imitation-innovation regime. These are:

$$\frac{\partial \mathbb{L}_{1,t+1}}{\partial u_{\mathrm{mi},t+1}} = \lambda \delta \sigma u_{\mathrm{mi},t+1}^{\sigma-1} s_{\mathrm{mi},t+1}^{1-\sigma} \frac{1}{\overline{A}_{t+1}} (\overline{A}_{t} - A_{t}) - w_{\mathrm{u},t+1} = 0;$$

$$\frac{\partial \mathbb{L}_{1,t+1}}{\partial u_{\mathrm{ni},t+1}} = \lambda \delta \gamma \phi u_{\mathrm{ni},t+1}^{\phi-1} s_{\mathrm{ni},t+1}^{1-\phi} A_{t} - w_{\mathrm{u},t+1} = 0;$$

$$\frac{\partial \mathbb{L}_{1,t+1}}{\partial s_{\mathrm{mi},t+1}} = \lambda \delta (1 - \sigma) u_{\mathrm{mi},t+1}^{\sigma} s_{\mathrm{mi},t+1}^{-\sigma} \frac{1}{\overline{A}_{t+1}} (\overline{A}_{t} - A_{t}) - w_{\mathrm{s},t+1} = 0;$$

$$\frac{\partial \mathbb{L}_{1,t+1}}{\partial s_{\mathrm{ni},t+1}} = \lambda \delta \gamma (1 - \phi) u_{\mathrm{ni},t+1}^{\phi} s_{\mathrm{ni},t+1}^{-\phi} A_{t} - w_{\mathrm{s},t+1} = 0.$$
(24)

Given that all the intermediate good producers are ex-ante identical, they face the same maximization problem. Thus, in equilibrium we have:

$$u_{\text{mi,t+1}} = u_{\text{m,t+1}}, \qquad u_{\text{ni,t+1}} = u_{\text{n,t+1}}, \qquad s_{\text{mi,t+1}} = s_{\text{m,t+1}} \qquad \text{and} \qquad s_{\text{ni,t+1}} = s_{\text{n,t+1}}.$$
 (25)

There is mass 1 of intermediate firms, so that labor market equilibrium condition is

$$S_{t+1} = s_{m,t+1} + s_{n,t+1},$$
 and $U_{t+1} = u_{m,t+1} + u_{n,t+1}.$ (26)

From the first-order conditions in the cluster of eqs. (24) and by using eq. (25), we get the relative demand curves for skilled and unskilled human capital in the imitation and innovation activities respectively as:

$$\frac{w_{\rm s,t+1}}{w_{\rm u,t+1}} = \frac{(1-\sigma)}{\sigma} \frac{u_{\rm m,t+1}}{s_{\rm m,t+1}}; \qquad \text{and} \qquad \frac{w_{\rm s,t+1}}{w_{\rm u,t+1}} = \frac{(1-\phi)}{\phi} \frac{u_{\rm n,t+1}}{s_{\rm n,t+1}}.$$
(27)

The equalization of the relative wage rate in eq. (27) implies:

$$\psi \; \frac{s_{\mathrm{m,t+1}}}{u_{\mathrm{m,t+1}}} = \frac{s_{\mathrm{n,t+1}}}{u_{\mathrm{n,t+1}}},\tag{28}$$

where, $\psi = \frac{\sigma(1-\phi)}{\phi(1-\sigma)} > 1$, by **A1**.

Accordingly, the demand levels for skilled and unskilled human capital in the imitation and innovation activities are worked out to be:¹⁹

$$s_{n,t+1} = \frac{\psi S_{t+1} - h(a_t) U_{t+1}}{\psi - 1}; \qquad s_{m,t+1} = \frac{h(a_t) U_{t+1} - S_{t+1}}{\psi - 1}; \\ u_{n,t+1} = \frac{\psi S_{t+1} - h(a_t) U_{t+1}}{(\psi - 1) h(a_t)}; \qquad u_{m,t+1} = \frac{\psi [h(a_t) U_{t+1} - S_{t+1}]}{(\psi - 1) h(a_t)}.$$
(29)

¹⁹Detailed mathematical derivations are provided in the cluster of eqs. ??-?? in Appendix A in pages ??-??.

where $h(a_t) = \left[\frac{(1-\sigma)\psi^{\sigma}(1-a_t)}{\gamma(1-\phi)(1+\bar{g})\bar{A}_t a_t}\right]^{\frac{1}{(\sigma-\phi)}}$, which is a decreasing function of the distance to frontier. That is, $h'(a_t) < 0.^{20}$

From eq. (29), the relative demands for skilled and unskilled human capital in the imitation and in the innovation activities are estimated to be:

$$\frac{s_{m,t+1}}{u_{m,t+1}} = \frac{h(a_t)}{\psi}; \qquad \qquad \frac{s_{n,t+1}}{u_{n,t+1}} = h(a_t). \tag{30}$$

3.4.2 Equilibrium

By equating the demand and supply curves of skilled and unskilled human capital, the equilibrium level of use of both types of inputs can be ascertained in the diversified regime. Furthermore, the equilibrium allocation of skilled and unskilled human capital in the imitation and innovation activities can also be derived.

First the cutoff level of cognitive ability above which an individual goes for education given that his/ her parent was educated or not is determined. Substituting eqs. (27) and (30) in eq. (8), we get,

$$\theta_{t+1}^{u} = \frac{2H}{1 - \frac{\phi}{(1-\phi)}h(a_{t})}; \qquad \qquad \theta_{t+1}^{s} = \frac{2H\frac{x_{u,t}}{x_{s,t}}}{1 - \frac{\phi}{(1-\phi)}h(a_{t})}.$$
(31)

From eq. (31), the proportion of unskilled human capital in the imitation-innovation regime is derived to be:²¹

$$U_{t+1} = \theta_{t+1}^{s} S_{t} + \theta_{t+1}^{u} U_{t} = \frac{2H(1-\phi) \left[U_{t} + \frac{x_{ut}}{x_{st}} S_{t} \right]}{\left[(1-\phi) - \phi h(a_{t}) \right]}.$$
(32)

Next, the equilibrium proportion of skilled human capital in the imitation-innovation regime can be derived from eq. (32) as:

$$S_{t+1} = 1 - U_{t+1} = 1 - \frac{2H(1-\phi)\left[U_t + \frac{x_{ut}}{x_{st}}S_t\right]}{\left[(1-\phi) - \phi h(a_t)\right]}.$$
(33)

Given the essentiality of both the inputs, the following conditions are needed as well:

$$U_{t+1} > 0 \Rightarrow [(1-\phi) - \phi h(a_t)] > 0;$$

$$S_{t+1} > 0 \Rightarrow 1 - \frac{2H(1-\phi) \left[U_t + \frac{x_{ut}}{x_{st}} S_t \right]}{[(1-\phi) - \phi h(a_t)]} > 0 \Rightarrow H < \frac{[(1-\phi) - \phi h(a_t)]}{2(1-\phi) \left[U_t + \frac{x_{ut}}{x_{st}} S_t \right]}.$$
(34)

The regularity condition in eq. (34) is not bounded.²² Next, some comparative dynamics analyses are also attempted. The change in the total stock of skilled and unskilled human capital as an economy progresses have been examined. Since, the equilibrium proportion of skilled and unskilled human capital are history

²⁰Detailed mathematical derivations are provided in the cluster of eqs. (??) in Appendix A on page ??.

²¹Detailed mathematical derivations are provided in the cluster of eqs. (??) in Appendix A on page ??.

²²By using **Lemma 3** on page 24, unboundedness of this condition has been shown in the cluster of eqs. (??) in Appendix A on page ??.



Figure 5: Diversified Regime – Skilled-Unskilled Human Capital and Allocation of it in Imitation and Innovation Activities

dependent, analytical solutions are not feasible. Therefore, numerical simulation has to be resorted to. The following specific parameter values have been assumed for the diversified regime:

Parameters	a(1)	U(1)
Values	0.5	0.65

Table 4: Specific Parameter Values for Numerical Simulation in the Imitation-Innovation Regime

These imply that the economy is neither sufficiently backward nor sufficiently advanced. It is characterized by intermediate values of the distance to frontier. Moreover, it neither has relatively high nor relatively low composition of skilled human capital. The parameter values also satisfy the regularity condition mentioned in **Lemma 3** in Subsection 3.4 on page 24.

The catch-up component is high for a technologically backward economy. As an economy progresses, its relative gap from the world technology frontier reduces. Consequently, the relative importance of imitation activity decreases and that of innovation activity increases. From **A1**, in equilibrium, the proportion of unskilled human capital falls and skilled human capital rises as an economy progresses, that is, $\frac{\partial U_{t+1}}{\partial a_t} < 0$ and $\frac{\partial S_{t+1}}{\partial a_t} > 0$, as is shown in Fig. 5a on page 22. These findings of the labor market equilibrium condition of the diversified regime are in line with Basu and Mehra [2014].

Lemma 1 Under A1,

- In the imitation-only regime, there exists a fixed composition of skilled and unskilled human capital. Moreover, the equilibrium proportion of skilled human capital is lower than unskilled human capital.
- In the innovation-only regime also there exists a constant composition of skilled and unskilled human capital. Additionally, the equilibrium proportion of skilled human capital is higher than unskilled

human capital in this regime.

• For a country which is in the imitation-innovation regime, the proportion of skilled human capital increases and unskilled human capital decreases as the country moves to the world technology frontier.

From the cluster of eqs. in (29) and eqs. (32) and (33), the equilibrium allocation of skilled and unskilled human capital can be ascertained in the imitation and in the innovation activities.²³ These will be:

$$\begin{split} s_{\mathrm{m,t+1}} &= \frac{2 \ H \ (1-\phi)[1+h(a_{\mathrm{t}})] \ \left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}}S_{\mathrm{t}}\right]}{(\psi-1) \left[(1-\phi) - \phi h(a_{\mathrm{t}})\right]} - \frac{1}{(\psi-1)};\\ u_{\mathrm{m,t+1}} &= \frac{2 \ H \ \psi \ (1-\phi)[1+h(a_{\mathrm{t}})] \ \left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}}U_{\mathrm{t}}\right]}{(\psi-1)h(a_{\mathrm{t}}) \left[(1-\phi) - \phi h(a_{\mathrm{t}})\right]} - \frac{\psi}{(\psi-1) \ h(a_{\mathrm{t}})};\\ s_{\mathrm{n,t+1}} &= \frac{\psi}{(\psi-1)} - \frac{2 \ H \ (1-\phi) \left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}}S_{\mathrm{t}}\right] \left[\psi + h(a_{\mathrm{t}})\right]}{(\psi-1) \ \left[(1-\phi) - \phi h(a_{\mathrm{t}})\right]}; \end{split}$$

$$u_{n,t+1} = \frac{\psi}{(\psi-1)h(a_t)} - \frac{2 H (1-\phi) \left[U_t + \frac{x_{ut}}{x_{st}} S_t \right] [\psi + h(a_t)]}{(\psi-1) h(a_t) [(1-\phi) - \phi h(a_t)]}.$$
(35)

Comparative dynamics have been carried out to capture the change in the allocation of skilled and unskilled human capital in both the imitation and the innovation activities as an economy progresses. By **Lemma 1**, the proportion of skilled (resp. unskilled) human capital increases (resp. decreases) as an economy progresses. By **A1**, innovation is more skilled human capital intensive. Therefore, innovation attracts more skilled human capital than imitation as the gap from the world technology frontier falls. Due to complementarity, unskilled human capital also shifts from imitation to innovation. This attracts even more skilled human capital into the innovation activity and the process goes on. Therefore, in equilibrium, both skilled and unskilled human capital increase in the innovation activity and decrease in the imitation activity, as is shown in Fig. 5b on page 22. That is, $\frac{d s_{m,t+1}}{d a_t} < 0$, $\frac{d u_{m,t+1}}{d a_t} < 0$, $\frac{d s_{n,t+1}}{d a_t} > 0$ and $\frac{d u_{n,t+1}}{d a_t} > 0$ which are analogues with Basu and Mehra [2014].²⁴

Lemma 2 Under A1,

In the imitation-innovation regime, the proportion of both skilled and unskilled human capital shift from the imitation activity to the innovation activity as an economy bridges its gap from the world technology frontier.

Next, the regularity conditions for the existence of positive amounts of both skilled and unskilled human capital in the imitation and innovation activities have been derived. That is, we need to have $s_{m,t+1} > 0$,

²³Detailed mathematical derivations are provided in the cluster of eqs. (??) in Appendix A on page ??.

²⁴Allocation of skilled and unskilled human capital in the imitation and innovation activities are also history dependent. Thus, for dynamics analysis, one needs to take the help of numerical simulation.

 $s_{m,t+1} < S_{t+1}, s_{n,t+1} > 0, s_{n,t+1} < S_{t+1}, u_{m,t+1} > 0, u_{m,t+1} < U_{t+1}, u_{n,t+1} > 0$ and $u_{n,t+1} < U_{t+1}$. This entails the following regularity condition:²⁵

$$\frac{h(a_{t})}{\psi} < \frac{S_{t+1}}{U_{t+1}} < h(a_{t}) \qquad [From eqs.(29)]$$

$$\Rightarrow \quad \frac{[(1-\phi)-\phi h(a_{t})]}{2 \ (1-\phi) \ [1+h(a_{t})] \ \left[S_{t} + \frac{x_{st}}{x_{ut}} \ U_{t}\right]} < H < \frac{\psi \ [(1-\phi)-\phi \ h(a_{t})]}{2 \ (1-\phi)[\psi + h(a_{t})] \ \left[S_{t} + \frac{x_{st}}{x_{ut}} \ U_{t}\right]}.$$

Intuitively this condition implies that, an economy specializes completely in the imitation activity if and only if $\frac{h(a_t)}{\psi} > \frac{S_{t+1}}{U_{t+1}}$. Further, the economy specializes in the innovation activity if and only if $\frac{S_{t+1}}{U_{t+1}} > h(a_t)$. If, however, the ratio of skilled to unskilled human capital lies within these bounds only then an economy performs both the imitation and the innovation activities, that is, economy is in the diversified regime. From eq. (??) in Appendix A, it is clear that $h(a_t)$ is a decreasing function of a_t . On the one hand, given any fixed composition of skilled-unskilled human capital (that is, with a fixed value of $\frac{S_{t+1}}{U_{t+1}}$), significantly technologically backward economies, (that is, economies with enough low a_t), specialize in the imitation activity, sufficiently technologically advanced economies (that is, economies with enough high a_t) specialize in the innovation activity and the intermediate economies perform both the activities. On the other hand, by **A1**, given any fixed distance to the frontier, an economy with significantly high (resp. low) composition of skilled to unskilled human capital ratio specializes in the innovation (resp. imitation) activity and the intermediate economies perform both the activities. In this analysis, the parametric value H represents the cost of education. A higher (resp. lower) H implies higher (resp. lower) cost of education and lower (resp. higher) equilibrium proportion of skilled human capital in the economy, entailing that the economy depends more on the imitation (resp. innovation) activity.

Lemma 3 Under A1,

• For technology improvement an economy performs both imitation and innovation activities if and only if

$$\frac{\left[(1-\phi)-\phi h(a_t)\right]}{2 \ (1-\phi) \ \left[1+h(a_t)\right] \ \left[S_t+\frac{x_{st}}{x_{ut}} \ U_t\right]} < H < \frac{\psi \ \left[(1-\phi)-\phi \ h(a_t)\right]}{2 \ (1-\phi)[\psi+h(a_t)] \ \left[S_t+\frac{x_{st}}{x_{ut}} \ U_t\right]};$$

• an economy specializes in imitation-only regime if and only if

$$\frac{\left[(1-\phi)-\phi h(a_t)\right]}{2 \ (1-\phi) \ \left[1+h(a_t)\right] \ \left[S_t+\frac{x_{st}}{x_{ut}} \ U_t\right]} > H;$$

• and an economy specializes in innovation-only regime if and only if

$$\frac{\psi \left[(1-\phi) - \phi \ h(a_t) \right]}{2 \ (1-\phi)[\psi + h(a_t)] \ \left[S_t + \frac{x_{st}}{x_{ut}} \ U_t \right]} < H.$$

²⁵Detailed mathematical derivations are provided in the cluster of eqs. ??-?? in Appendix A on page ??.

3.4.3 Growth Rate

We now derive the growth rate for an economy which is in the diversified regime. From eq. (3), we get,²⁶

$$g_{t+1} = \lambda \left[u_{m,t+1}^{\sigma} s_{m,t+1}^{1-\sigma} \frac{1}{\overline{A}_{t+1}} \left(\frac{1-a_{t}}{a_{t}} \right) + \gamma u_{n,t+1}^{\phi} s_{n,t+1}^{1-\phi} \right]$$
$$= \lambda \left[\underbrace{\left(\underbrace{u_{m,t+1}}_{s_{m,t+1}} \right)^{\sigma}}_{+ve} \underbrace{s_{m,t+1}}_{-ve} \frac{1}{\overline{A}_{t+1}} \underbrace{\frac{(1-a_{t})}{a_{t}}}_{-ve} + \gamma \underbrace{\left(\underbrace{u_{n,t+1}}_{s_{n,t+1}} \right)^{\phi}}_{+ve} s_{n,t+1}}_{+ve} \right]$$
$$= \lambda \gamma (1-\phi) h^{-\phi}(a_{t}) \left[1 - 2H \left(S_{t} + \frac{x_{st}}{x_{ut}} U_{t} \right) \right].$$
(36)

Thus a positive growth rate entails that $H < \frac{1}{2\left(S_t + \frac{x_{st}}{x_{ut}}U_t\right)}$. However, this condition is not bounded. From eq. (36), it is easy to see that the growth rate of an economy in the diversified regime depends on the relative intensity of unskilled and skilled human capital in the imitation and innovation activities, the allocation of skilled human capital in these two activities and the distance of an economy from the world technology frontier. Combining eq. (30) with eq. (??) in Appendix A, we get that the relative intensity of unskilledskilled human capital in these two activities depend positively on the distance to frontier. Further, the allocation of skilled human capital in the imitation (resp. innovation) activity depends negatively (resp. positively) on the distance to frontier. Finally, we have the result that the relative gap from the frontier declines as an economy progresses. It is the interaction of all of these factors that determine the growth rate of an economy in the diversified regime. Eq. (36) reveals that the growth rate is history dependent. Therefore, numerical simulations are required to characterize the growth path. From Fig. 6a on page 26, it is clear that the growth rate initially falls and thereafter, rises as an economy progresses. That is, it is a U-shaped growth curve. As an economy shifts from the imitation-only regime to the diversified regime, the scope for both imitation and innovation is low. First, the scope of imitation is low since the advantage of backwardness is falling as an economy progresses. Second, the scope of innovation is low in the initial stages of development of the diversified regime, since the technology level is not sufficiently high enough. Further, from **Lemma 1**, the relative supply of skilled human capital is also low. This causes the falling part of the growth path in the diversified regime. As the time progresses, the opportunity for innovation rises and again from **Lemma 1**, the proportion of skilled human capital rises that leads to an incrementally higher rate of growth. Due to the consideration of the disadvantage of backwardness, in the initial stages of the diversified regime growth rate falls which is in contrast of Vandenbussche et al. [2006], Aghion et al. [2009] and Basu and Mehra [2014].

Proposition 1 Under A1,

• In the imitation-only regime, the growth rate falls as an economy progresses.

²⁶Detailed mathematical derivations are provided in the cluster of eqs. (??) in Appendix A on page ??.



Figure 6: Diversified Regime – Growth Rate and Wage Paths of Skilled and Unskilled Human Capital

- In the diversified regime, the growth rate initially falls and thereafter, rises as an economy progresses, that is, there exists a U-shaped growth path.
- In the innovation-only regime, there exists a constant growth rate.

3.4.4 Wage Rate

The discussion now shifts to the dynamics paths of the wage rate of skilled and unskilled workers as an economy bridges its gap from the world frontier. Substituting eq. (30) in the cluster of eqs. (24), we get,²⁷

$$\begin{split} w_{\mathrm{u, t+1}} &= \lambda \delta \gamma \phi h^{1-\phi}(a_{\mathrm{t}}) a_{\mathrm{t}} \overline{A}_{\mathrm{t}}; \\ w_{\mathrm{s, t+1}} &= \lambda \delta \gamma (1-\phi) h^{-\phi}(a_{t}) a_{\mathrm{t}} \overline{A}_{\mathrm{t}}. \end{split}$$

In the diversified regime, as an economy progresses, the relative importance of innovation increases and that of imitation falls. From A1, the marginal productivity of skilled human capital increases and that of unskilled human capital decreases and so the wage rate of skilled workers rises while that of unskilled workers falls. Consequently, the wage gap between skilled and unskilled workers grows, as is also demonstrated in Fig. 6b on page 26. These findings are analogues with Basu and Mehra [2014] however in contrast with Maoz and Moav [1999].

Proposition 2 Under A1,

- In the imitation-only regime, the wage rates of skilled and unskilled workers fall as an economy progresses. There exists a constant level of wage inequality between skilled and unskilled groups.
- In the diversified regime, the wage rate of skilled workers rises and that of unskilled workers falls as an economy steps forward toward the world technology frontier. Consequently, the wage inequality between skilled and unskilled workers rises.

²⁷Detailed mathematical derivations are provided in the cluster of eqs. (??) in Appendix A on page ??.



Figure 7: Steady State – Specialized Regimes

• In the innovation-only regime, the wage rates of skilled and unskilled workers rise as an economy progresses. However, there exists a constant level of wage inequality between skilled and unskilled workers.

3.5 Steady State

This subsection incorporates a discussion on the long run equilibrium condition of an economy. We attempt to find an answer to the question: as the time progresses does an economy converge its gap from the world technology frontier, depending on its distance to the frontier? The definition of growth rate can be specified as:

$$g_{t+1} = \frac{A_{t+1} - A_t}{A_t} = \frac{A_{t+1}}{A_t} - 1$$

$$= \frac{A_{t+1}}{\overline{A}_{t+1}} \frac{\overline{A}_t}{A_t} (1 + \overline{g}) - 1$$

$$= \frac{a_{t+1}(1 + \overline{g})}{a_t} - 1$$

$$\Rightarrow a_{t+1} = \frac{(1 + g_{t+1})}{(1 + \overline{g})} a_t$$
(37)

When the growth rate of an economy is higher than the growth rate of the world leader, it will be able to converge to the frontier, and in the long run, it will catch up with the frontier technology level.²⁸ Numerical simulation also corroborates this. From Fig. 7a on page 27, Fig. 7b on page 27 and Fig. 8 on page 28, it is clear that as an economy progresses it closes its distance from the world technology frontier. In the long run, all the economies will converge to the world technology level. Steady state implies that a_t will converge to a^* , that is, $a_t \to a^*$ and the growth rate of the economy will converge to g^* , that is, $g_t \to g^*$.

²⁸The economy will catch up if next period's distance to frontier (that is, a_{t+1}) is higher than the earlier period's distance from the world technology frontier (that is, a_t).



Figure 8: Steady State – Diversified Regime



Figure 9: Upward and Downward Mobility – Diversified Regime

Therefore, eq. (37) implies that either $g^* = \bar{g}$ or $a^* = 0$. Mathematically the possibility of the outcome of $a^* = 0$ is coming out. However, it is not a economically sensible outcome. As it entails a zero output and consumption. Therefore, the focus of study is limited to the case of $g^* = \bar{g}$. This implies that in the long run all the economies will grow at the same rate, that is, there exists **absolute convergence** of the economies in the long run. This implies that growth enhancing education policy helps to converge to the world technology frontier even if capital market is imperfect which is similar with Basu and Mehra [2014].

Proposition 3 In the long-run all the economies will converge to the world technology frontier irrespective of its distance to frontier. Moreover, in the steady state, all the economies will grow at the same rate.

3.6 Intergenerational Mobility

This subsection presents an analysis of upward and downward mobility of individuals. Upward mobility is implied by the phenomenon that an individual works as a skilled worker given that his/ her parent was unskilled, that is,

$$UM_{t+1} = U_t(1 - \theta_{t+1}^u),$$

where, UM_{t+1} measures upward mobility in period (t+1). It captures the probability of moving from low equilibrium to high equilibrium; that is, the proportion of skilled human capital (that is, $(1 - \theta_{t+1}^u)$) whose parents were unskilled (that is, U_t). It is a cross product of today's skilled people with earlier period's



Figure 10: Upward and Downward Mobility – Specialized Regimes

unskilled individuals. Next, downward mobility implies that parent was skilled but child is working as an unskilled worker, that is,

$$\mathrm{DM}_{\mathrm{t+1}} = S_{\mathrm{t}}\theta_{\mathrm{t+1}}^{\mathrm{s}},$$

where DM_{t+1} measures downward mobility in period (t+1). Clearly, it is the opposite of upward mobility. It captures the probability of moving from a high to low equilibrium. It is a cross product of the proportion of today's unskilled human capital (that is, θ_{t+1}^s) whose parent was skilled (that is, S_t). Intergenerational mobility helps us to analyze the correlation between cognitive ability and income of an individual. Low mobility implies that individuals whose parents have a high (resp. low) income have high chance of working as skilled (resp. unskilled). The focus of our analysis is to identify the dynamic paths of upward and downward mobility in the diversified regime.

From **Proposition 2**, in the diversified regime, the wage gap between skilled and unskilled human capital rises as an economy progresses technologically. Individuals whose parents were unskilled and left a lesser amount of bequest have a lower probability of becoming educated than the individuals whose parents were skilled. Subsequently, both upward and downward mobility fall as an economy progresses (as is shown in Fig. 9 on page 28). As the gap from the world frontier falls, the probability of shifting from low equilibrium to high equilibrium as well as from high equilibrium to low equilibrium fall in the diversified regime. In the specialized regimes, from Lemma 1, there exists a fixed proportion of the composition of human capital and from **Proposition 2**, there exists constant wage inequality. This implies a constant level of upward and downward mobility in the specialized regimes, as are also shown in Fig. 10a on page 29 and Fig. 10b on page 29. To conclude, if parents were educated (skilled) the probability of the children being educated weakly rises and if parents were unskilled then the opportunity for the children becoming educated weakly falls. That is, education becomes more correlated with parental income and less correlated with child's cognitive ability. It implies that as wage inequality between groups weakly rises mobility falls. There exists a weakly inverse relation between intergenerational mobility and wage inequality. This implies that the endogenous technology formation leads to a opposite findings of Maoz and Moay [1999] that along the growth path education becomes more correlated with the cognitive ability and the dependence of the



Figure 11: Imitation-Only Regime – Life time Utility Gaps within Skilled and Unskilled Human Capital

initial condition falls.

Proposition 4 Under A1,

- In the imitation-only and innovation-only regimes there exists a constant level of upward and downward mobility as an economy bridges the gap from the world technology frontier.
- In the diversified regime, both upward and downward mobility fall as the economy progresses technologically.

3.7 Life Time Utility

In this subsection, first, the average life time utility of skilled and unskilled workers given their parental income have been determined. Second, within skilled and unskilled groups life time utility gap due to parental income differences have been worked out. Finally, within skilled group life time utility gap due to differences in the cognitive ability among individuals has been ascertained. From eq. (6), the life time utility of skilled and unskilled workers are defined as a function of consumption levels in both the periods of life and the level of bequest that they leave for their children. This, in turn, depends on the wage income of that individual, on the level of bequest that he/ she receives from parent and on the cost of education (if he/ she is skilled). The level of bequest received and the cost of education respectively vary among individuals depending on the parental income and cognitive ability. The average cost of education of individuals whose parents were skilled is denoted by E_{t+1}^{ss} and can be expressed as:

$$E_{t+1}^{ss} = \frac{\frac{H w u_t}{\theta} |_{\theta=1} + \frac{H w u_t}{\theta} |_{\theta=\theta_{t+1}^s}}{2}.$$
(38)

Similarly, the average cost of education of individuals whose parents were skilled represented by E_{t+1}^{su} will be:

$$E_{t+1}^{su} = \frac{\frac{H \ wu_t}{\theta}}{2} \left|_{\theta=1} + \frac{H \ wu_t}{\theta} \right|_{\theta=\theta_{t+1}^{u}}.$$
(39)



Figure 12: Diversified Regime – Life time Utility Gaps within Skilled and Unskilled Human Capital

Thus, E_{t+1}^{ss} and E_{t+1}^{su} are respectively defined as the weighted average of the cost of education of an individual with highest and lowest cognitive ability who go for education depending on whether parents were skilled or unskilled.

Accordingly, the average life time utility of a skilled individual whose parents were skilled and unskilled respectively denoted by W_{t+1}^{ss} and W_{t+1}^{su} . Specifically, from eqs. (6), (38) and (39), these are expressed as:

$$W_{t+1}^{ss} = \left[\frac{ws_t}{2} - E_{t+1}^{ss}\right] \frac{ws_{t+1}}{2};$$
$$W_{t+1}^{su} = \left[\frac{wu_t}{2} - E_{t+1}^{su}\right] \frac{ws_{t+1}}{2}.$$

Assuming the same level of bequest, the consumption of the k^{th} individual in the first period of life is positively related with the cognitive ability of an individual (since the first period consumption is the gap between the level of bequest received from parents and the cost of education.)

Similarly, the average life time utility of an unskilled individual whose parent was skilled and unskilled respectively are denoted by W_{t+1}^{us} and W_{t+1}^{uu} . Specifically, from eq. (6), these will be:

$$W_{t+1}^{us} = \frac{ws_t}{2} \frac{wu_{t+1}}{2};$$
$$W_{t+1}^{uu} = \frac{wu_t}{2} \frac{wu_{t+1}}{2}.$$

3.7.1 Life Time Utility Gap Due to Parental Income Differences

Life time utility gaps within skilled and unskilled workers have been defined. First, the life time utility gap due to parental income differences has been analyzed.

$$\operatorname{Win}_{t+1}^{s} = \frac{W_{t+1}^{ss}}{W_{t+1}^{su}}; \qquad \qquad \operatorname{Win}_{t+1}^{u} = \frac{W_{t+1}^{us}}{W_{t+1}^{uu}}$$

where $\operatorname{Win}_{t+1}^{s}$ and $\operatorname{Win}_{t+1}^{u}$ respectively measure life time utility gap within skilled and unskilled workers due to differences in the parental education level. From **Proposition 2**, in the diversified regime, as an



Figure 13: Innovation-Only Regime – Life time Utility Gap within Skilled and Unskilled Human Capital

economy moves toward the world technology frontier, the wage gap between skilled and unskilled workers rises. Therefore, the gap between the level of bequest that a skilled as well as unskilled individuals obtain from parents due to difference in the parental income rise. This leads to a higher life time utility gap within skilled as well as unskilled workers as is shown in Fig. 12a on page 31. From **Proposition 2**, in the specialized regimes, there exists a constant wage inequality between skilled and unskilled workers. This implies that there exists a constant gap between the levels of bequest due to difference in parental income. This leads to a constant life time utility gap within skilled as well as unskilled workers, as can be seen in Fig. 11a on page 30 and Fig. 13a on page 32.

3.7.2 Life Time Utility Gap Due to Cognitive Ability Differences

Now, the life time utility gap among skilled workers due to differences in the cognitive ability has been discussed. We have,

$$\operatorname{Win}_{t+1}^{\theta s} = \frac{W_{t+1}^{ss}|_{\theta=1}}{W_{t+1}^{ss}|_{\theta=\theta^s}}; \qquad \qquad \operatorname{Win}_{t+1}^{\theta u} = \frac{W_{t+1}^{su}|_{\theta=1}}{W_{t+1}^{su}|_{\theta=\theta^u}},$$

where $\operatorname{Win}_{t+1}^{\theta s}$ and $\operatorname{Win}_{t+1}^{\theta u}$ respectively measure life time utility gap due to cognitive ability differences among skilled workers even if all of their parents were skilled and unskilled. These capture the life time utility gap due to difference in cognitive ability through the lifetime utility gap of skilled workers between the highest and lowest cognitive ability. By **Lemma 1**, as an economy progresses, skilled human rises in the diversified regime. This, in turn, implies that individuals with relatively low cognitive ability now become educated. As a result, the education cost gap among skilled human capital rises irrespective of their parental income levels. This leads to a higher wealth inequality within skilled human capital due to difference in cognitive ability. To conclude, in the diversified regime, due to difference in cognitive ability, lifetime utility gap within skilled workers rises irrespective of their parental education status as an economy progresses, this is depicted in Fig. 12b on page 31. Moreover, from eq. (8), we know that $\theta_{t+1}^s < \theta_{t+1}^u$. This implies that the cut off cognitive ability above which an individual goes for education takes lower



Figure 14: Imitation-Only Regime – Comparative Static wrt Cost of Education

value if parents worked as skilled workers. Consequently, children of skilled parents work as skilled even with relatively low cognitive ability than individuals with unskilled parents. Therefore, among skilled human capital, the life time utility gap is high due to difference in cognitive ability for the individuals whose parents were skilled than for those whose parents were unskilled. Given that there exists a fixed composition of human capital in the specialized regimes, there also exists constant level of life time utility gap within skilled workers due to difference in cognitive ability. This is true for both the cases – where parents were either skilled or unskilled, as can be seen in Fig. 11b on page 30 and Fig. 13b on page 32.

Proposition 5 Under A1,

- In the imitation-only and in the innovation-only regimes, there exists a constant level of life time utility gap within skilled as well as unskilled workers due to parental income differences. Moreover, there exists a constant level of life time utility gap within skilled workers due to cognitive ability differences, in the specialized regimes.
- In the imitation-innovation regime, the life time utility gaps within skilled and within unskilled workers rise due to parental income differences. The life time utility gap within skilled workers also rises on account of cognitive ability differences. However, the lifetime utility gap within skilled workers due to cognitive ability differences is higher if parents were skilled than when they were unskilled workers.

3.8 Comparative Dynamics Analysis w.r.t Cost of Education

A comparative dynamic analysis is done to analyze the impact of increment in the cost of education (captured by a change in the parametric value H) on the composition of human capital and on the growth rate of the economy. Due to a parametric positive change in the cost of education, the income of an individual who works as skilled decreases whereas income of the individual who works as unskilled remains unchanged. Therefore, the increment in the cost of education reduces the proportion of skilled human capital and increases that of unskilled human capital, irrespective of the economy's distance to the



Figure 15: Innovation-Only Regime - Comparative Static wrt Cost of Education



Figure 16: Diversified Regime – Comparative Static wrt Cost of Education

frontier, as are shown in Fig. 14a on page 33, in Fig. 15a on page 34 and in Fig. 16a on page 34. By A1, imitation depends more on unskilled human capital. Therefore, a rise in the cost of education raises the proportion of unskilled human capital and consequently, the growth rate of an economy which is in the imitation-only regime. This implies that unskilled human capital is growth enhancing in the imitation-only regime. Due to the reduction in the proportion of skilled human capital by A1, the growth rate falls in the innovation-only regime, as is illustrated in Fig. 15b on page 34. This, in turn, implies that skilled human capital is growth enhancing in the innovation-only regime, as is illustrated in Fig. 15b on page 34. This, in turn, implies that skilled human capital is growth enhancing in the innovation-only regime. In the diversified regime, by A1 and Lemma 2, both skilled and unskilled human capital shift from innovation to imitation activities and consequently, the growth rate falls, as are shown in Fig. 16b on page 34, in Fig. ?? on page ?? and in Fig. ?? on page ??. Thus, skilled human capital is growth enhancing in the innovation regime.

Proposition 6 Under A1,

- In the imitation-only regime, unskilled human capital is growth enhancing.
- In the imitation-innovation and innovation-only regimes, skilled human capital is growth enhancing.

4 Conclusion

Technological progress is assumed to be dual phenomenon – either imitate from the world technology frontier or innovate new knowledge. Innovation depends on its own technology. Along with the advantage of backwardness there also exists disadvantage of backwardness as mentioned by Vandenbussche et al. [2006] and Howitt [2000]. A skilled biased endogenous growth model, with complete absence of credit market (like Maoz and Moay [1999]) along with the assumption that different types of human capital are efficient in different activities (similar to Vandenbussche et al. [2006], Aghion et al. [2009] and Basu and Mehra [2014]) has been considered for this analysis. By utilizing this structure this study ables to provide a theoretical justification of the existing empirical findings of the existence of the high inequality and low mobility in the process of development. It is shown that there exists a U-shaped growth path depending on the economy's distance to frontier. Additionally, by applying a growth enhancing education policy, all the economies would converge to the world technology frontier. Moreover, in the diversified regime, the probability of becoming rich (resp. poor) given that the parent was rich (resp. poor) rises as an economy progresses. This implies that along the growth path, the correlation between income and cognitive ability falls and the importance of the initial conditions rise. Moreover, between skilled and unskilled group wage inequality rises in the diversified regime as an economy bridges its gap from the world technology frontier. However, there prevails a constant between group wage inequality in the specialized regimes. This entails that there exists a positive correlation between equality and intergenerational upward and downward mobility. To conclude it is shown that unskilled human capital is the main source of growth for the specialized backward economy and skilled human capital is the driving factor for growth for the diversified and specialized advanced economy.

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